Hydrodynamic Evolution from Flux Tube Initial Conditions in Ultrarelativistic Heavy Ion Collisions

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We work very actively on hydro for pp@LHC ...
but this talk: RHIC applications, as warm up exercise

Contents

1 Introduction

After one decade of RHIC experiments (heavy ion, pp, and dAu scattering, up to 200 GeV) it seems that

> heavy ion collisions produce matter
> which expands as an almost ideal fluid

- ▷ mainly because azimuthal anisotropies can be explained on the basis of ideal hydrodynamics (mass splitting etc)

But: there is not (yet) THE hydro approach
compatible with ALL RHIC data

all rapidities, all centralities ...

- **Basic "soft physics" RHIC data (only AuAu ≥ 200 distributions)**

  - **Particle yields and eta distributions**
    - STAR and PHENIX
      average yields and mean pt of pions, kaons, protons, lambdas, xis vs centrality
    - BRAHMS
      eta distr for different centralities 0-5% 5-10% 10-20% 20-30% 30-40% 40-50%
      rapidity distr of pions, kaons, protons(central)
      mean pt vs rapidity of pions, kaons (central)

  - **pt spectra**
    - PHOBOS: pt distributions of charged particles at centralities 0-6%, 6-15%, ..., 45-50%
    - BRAHMS: pt distributions of pions, kaons, protons at given rapidity (central)
    - PHENIX: pt distributions of pions, kaons, protons for different centralities:
      0-5%, 5-10%, 10-20%, 20-30%, 30-40%, 40-50%, 50-60%, 60-70%, 70-80%, 80-92%
    - STAR: mt distributions of pions, kaons, protons for different centralities
      0-5%, 5-10%, 10-20%, 20-30%, 30-40%, 40-50%, 50-60%, 60-70%, 70-80 %
    - STAR pt distributions of strange baryons for different centralities:
      0-5%, 10-20%, 20-40%, 40-60%, 60-80%

  - **v2:**
    - PHOBOS: v2 vs eta for different centralities: MB, 3-15, 15-25, 25-50, 0-40
      v2 vs centrality v2 vs pt of charged particles, 0-50
    - STAR v2 vs pt of pi, K, prt for different centralities
      MB, 0-5, 20-30, 40-50; Λ and K_s 10-40, 40-80
    - PHENIX v2 vs pt of π, K, p for 0-60, 20-60

We will present a "realistic" treatment of the hydrodynamic evolution of ultrarelativistic heavy ion collisions, based on the following features:

- ▷ initial conditions obtained from a flux tube approach (EPOS),
  - compatible with the string model used since many years for elementary collisions (electron-positron, proton proton),
  - and the color glass condensate picture;
- ▷ consideration of the possibility to have a (moderate) initial collective transverse flow;
- ▷ event-by-event procedure,
  - taking into the account the highly irregular space structure of single events,
  - leading to so-called ridge structures in two-particle correlations;
- ▷ core-corona separation,
  considering the fact that only a part of the matter thermalizes;

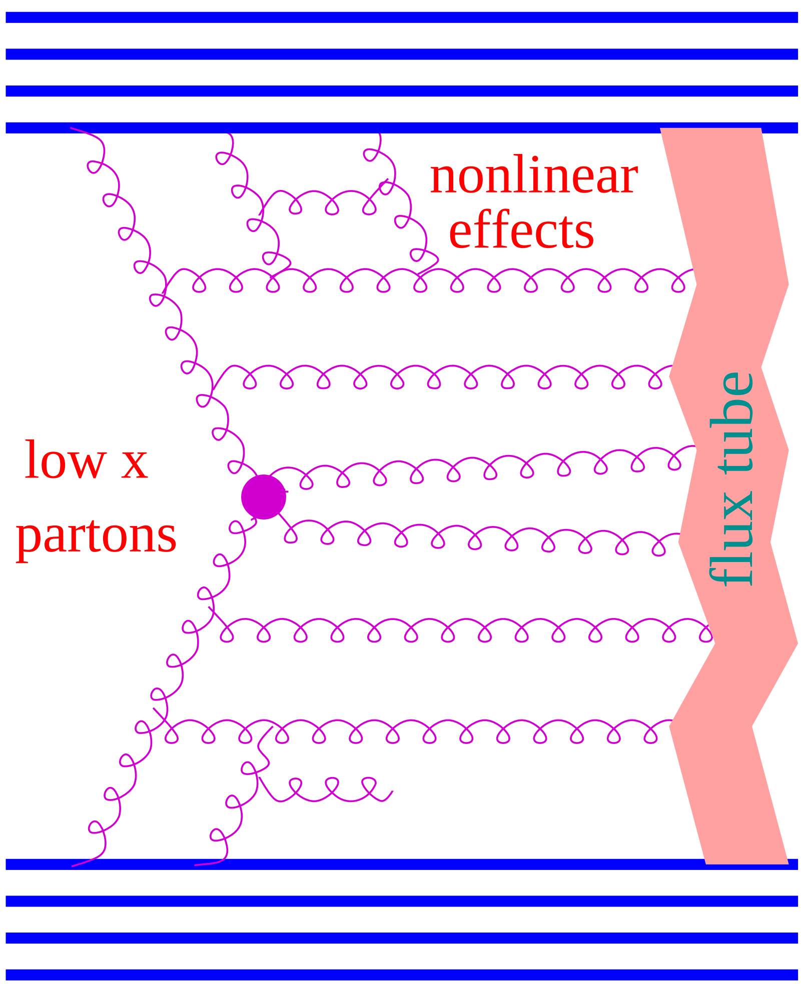
- ▷ use of an efficient code for solving the hydrodynamic equations in 3+1 dimensions, including the conservation of baryon number, strangeness, and electric charge;
- ▷ employment of a realistic equation-of-state, compatible with lattice gauge results – with a cross-over transition from the hadronic to the plasma phase;
- ▷ use of a complete hadron resonance table, making our calculations compatible with the results from statistical models;
- ▷ hadronic cascade procedure after hadronization from the thermal system at an early stage.

These features are not new!

New is the attempt to put all these elements into a single approach,
bringing together topics which are often discussed independently
like statistical hadronization, flow features, saturation, the string model, ...

2 Flux tubes

AA scattering - even pp: **many elementary collisions happening in parallel**
elementary scattering = "parton ladder"



- Parton evolutions from the projectile and the target side towards the center (small x)
- Evolution is governed by an evolution equation, in the simplest case according to DGLAP.
- Parton ladder may be considered as a quasi-longitudinal color field, a so-called "flux tube", conveniently treated as a relativistic string.
- The intermediate gluons are treated as kink singularities in the language of relativistic strings, providing a transversely moving portion of the object.
- This flux tube decays via the production of quark-antiquark pairs, creating in this way fragments – which are identified with hadrons

Quantum mechanical treatment of multiple scattering is quite involved!

(when the energy sharing between the parallel scatterings is taken into account)

Details:

Parton-based Gribov-Regge Theory, H. J. Drescher, M. Hladik, S. Ostapchenko, T.Pierog, and K. Werner, Phys. Rept. 350 (2001) 93-289;
Parton ladder splitting and the rapidity dependence of pt spectra in dAu collisions at RHIC, K. Werner, F.M. Liu, T. Pierog, Phys. Rev. C 74, 044902 (2006)

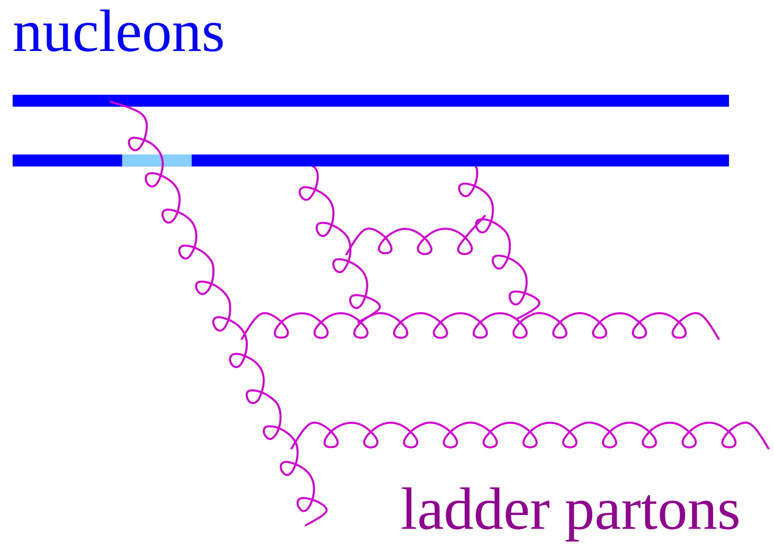
- ▷ Based on cutting rule techniques, one obtains partial cross sections for exclusive event classes,
- ▷ which are then simulated with the help of Markov chain techniques.

"parton ladder" is meant to contain two parts:

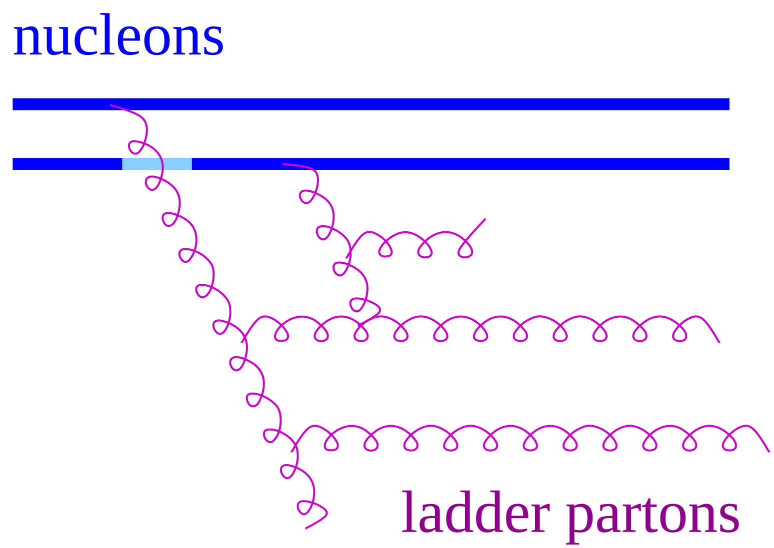
- ▷ the hard one, as discussed above (following an evolution equation),
- ▷ and a soft one, which is a purely phenomenological object, parametrized in Regge pole fashion

At high energies, non-linear effects:

Elastic "rescattering" of a ladder parton



Inelastic "rescattering" of a ladder parton



Elastic rescattering: parametrize a parton ladder [1] (computed on the basis of DGLAP) as

$$\alpha(x^+x^-)^\beta,$$

x^+ and x^-: momentum fractions of the “first” ladder partons on both sides (which initiate the parton evolutions).

The parameters α and β depend on the cms energy $\sqrt{s}$

To mimic the reduction (screening) of the expressions $\alpha(x^+x^-)^\beta$, use

$$\alpha(x^+)^{\beta+\varepsilon_P}(x^-)^{\beta+\varepsilon_T},$$

where the values of the positive numbers $\varepsilon_{P/T}$ will increase with the nuclear mass number and $\log s$.

[1] to be more precise: the imaginary part of the corresponding amplitude in impact parameter space

This additional exponent has very important consequences:

- ▷ it will reduce substantially the increase of both cross sections and multiplicity with the energy,
- ▷ having thus a similar effect as introducing a saturation scale.

Inelastic rescatterings:

- ▷ provide several ladders close to the projectile (or target) side, which are close to each other in space,
- ▷ treated via an enhancement of remnant excitations (not discussed here)

Parton ladder = elementary flux tube = classical string

The relativistic classical string picture is very attractive,
because its dynamics (Lagrangian) is essentially derived from general principles as covariance and gauge invariance

Simplest possible string: a two-dimensional surfaces $X(\alpha, \beta)$ in 3+1 dimensional space-time, with piecewise constant initial conditions,

$$V(\alpha) \equiv \frac{\partial X}{\partial \beta}(\alpha, \beta = 0) = V_k, \text{ in } [\alpha_k, \alpha_{k+1}],$$

referred to as kinky strings.

The dynamics is governed by the Nambu-Goto string action

Mapping partons onto strings:

- we identify the ladder partons with the kinks of a kinky string,
- such that the length of the α-interval is given by the parton energies E_k,
- and the kink velocities are just the parton velocities, p_k^μ / E_k.

The string evolution is then completely given by these initial conditions

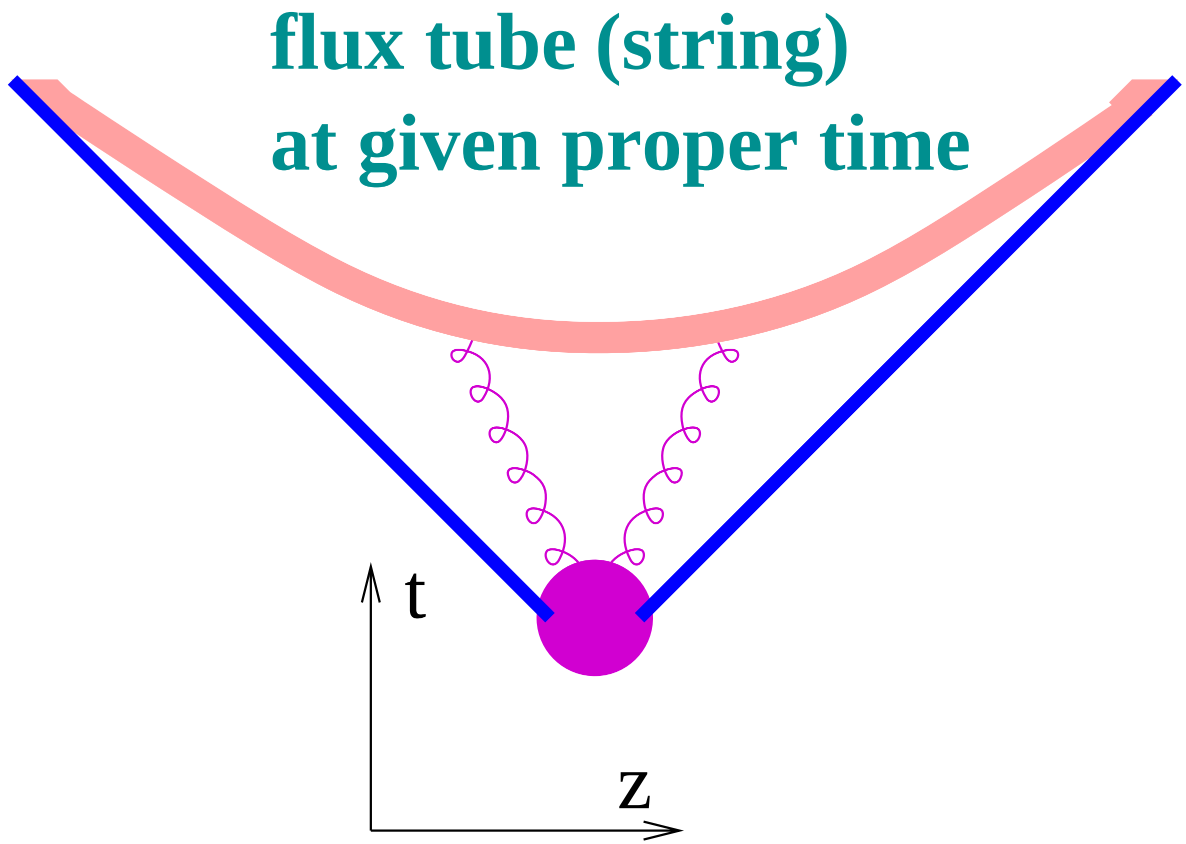
$$X(\alpha,\beta) = X_0 + \frac{1}{2}\left[\int_{\alpha-\beta}^{\alpha+\beta} V\left(\xi\right) d\xi\right].$$

String at a given proper time τ_0:

intersection of the string surface $X(\alpha, \beta)$ with the hypersurface corresponding to constant proper time: $\tau = \tau_0$.

simplified picture
in $z - t$ space

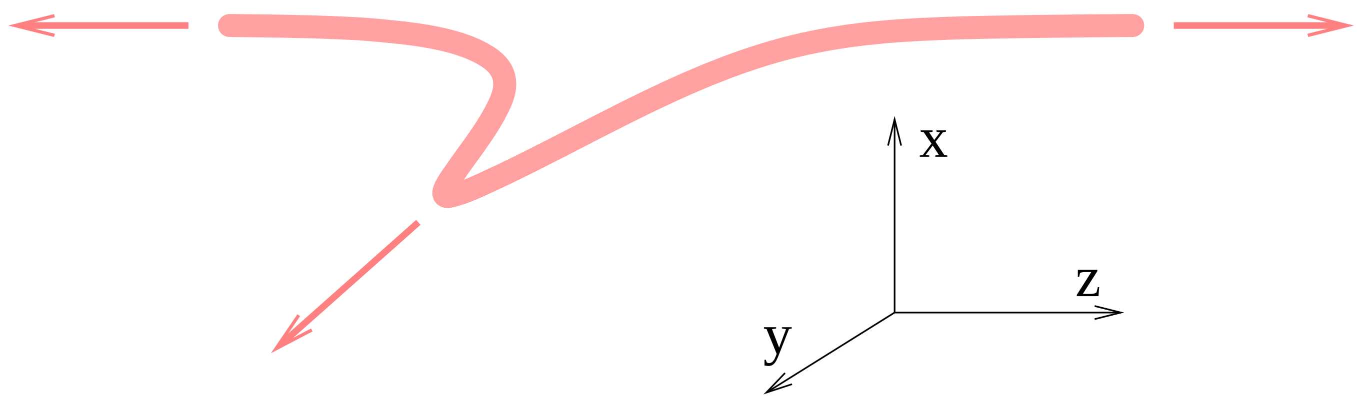
(in reality – and in our calculations, all three space dimensions are important, due to the transverse motion of the kinks)



Space components of the string in $\mathbb{R}^3$ space :

mainly longitudinal object (here parallel to the z-axis)

but due to the kinks there are string pieces moving transversely (in y-direction in the picture).



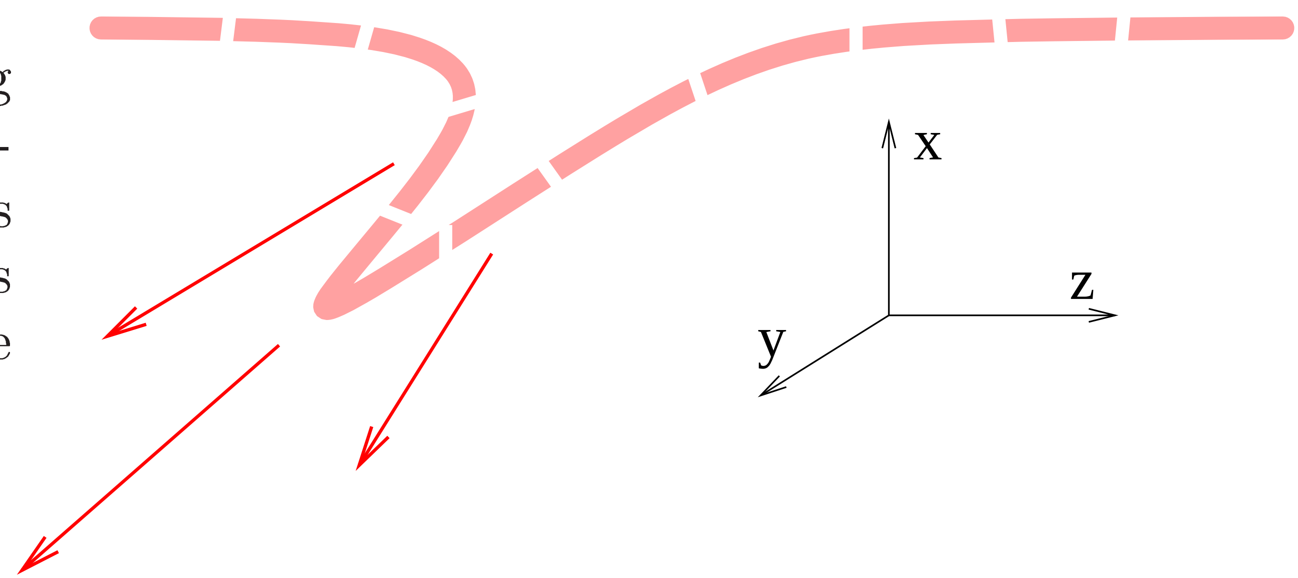
But despite these kinks, most of the string carries only little transverse momentum!

In case of elementary reactions

hadron production is realized via string breaking,
string fragments are identified with hadrons.

- the string breaks within an infinitesimal area dA on its surface with a probability proportional to this area, $dP = p_B\, dA$ (area law)
- The flavor dependence of the $q - \bar{q}$ or $qq - \overline{qq}$ string breaking is given by probabilities $\exp(-\pi m_q^2/\kappa)$,

After breaking, the string pieces close to a kink constitute the jets of hadrons (arrows) whose direction is mainly determined by the kink-gluon.

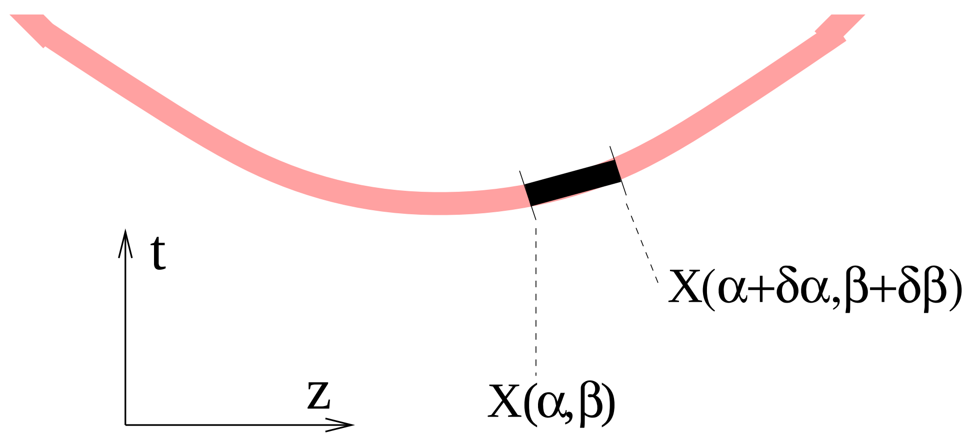


Heavy ion collisions or very high energy proton-proton scattering:

the procedure has to be modified,
since the density of strings will be so high
that they cannot possibly decay independently

We split each string into a sequence of string segments, corresponding to widths $\delta\alpha$ and $\delta\beta$ in the string parameter space

Picture is schematic: the string extends well into the transverse dimension, correctly taken into account in the calculations



One distinguishes between

- string segments in dense areas (more than some critical density ρ_0 of segments per unit volume) => **core**
- from those in low density areas => **corona**

Excluded from the core:

- String segments with large transverse momentum, close to a kink (no energy loss for the moment)
- Remnant baryons

For core part, energy momentum tensor and the flavor flow vector at some position x at initial proper time $\tau = \tau_0$:

$$T^{\mu\nu}(x) = \sum_i \frac{\delta p_i^\mu \delta p_i^\nu}{\delta p_i^0} g(x - x_i),$$

$$N_q^\mu(x) = \sum_i \frac{\delta p_i^\mu}{\delta p_i^0} q_i \, g(x - x_i),$$

$q \in u, d, s$: net flavor content of the string segments

$\delta p = \left\{ \frac{\partial X(\alpha,\beta)}{\partial \beta} \delta\alpha + \frac{\partial X(\alpha,\beta)}{\partial \alpha} \delta\beta \right\}$: four-momenta of the segments.

g: Gaussian smoothing kernel with a transverse width $\sigma_\perp = 0.25$ fm.

The Lorentz transformation into the comoving frame provides the energy density ε and the flow velocity components v^i.

3 Hydrodynamic evolution, realistic EoS

Core evolves according to the equations of hydrodynamics:

local energy-momentum conservation

$$\partial_\mu T^{\mu\nu} = 0, \quad \nu = 0, ..., 3\,,$$

and the conservation of net charges,

$$\partial N_k^\mu = 0, \quad k = B, S, Q\,,$$

with B, S, and Q referring to respectively baryon number, strangeness, and electric charge.

Here: ideal hydrodynamics:

$$T^{\mu\nu} = (\epsilon + p)\, u^\mu u^\nu - p\, g^{\mu\nu}\,, \quad N_k^\mu = n_k u^\mu,$$

where u is the four-velocity of the local rest frame (unknowns: $\varepsilon(x)$, $u(x)$, $n_k(x)$.

Crucial ingredient is the equation of state, which closes the set of equations by providing $p(\varepsilon)$

The equation-of-state should fulfill the following requirements:

- ▷ average flavor conservation, using chemical potentials μ_B, μ_S, μ_Q;
- ▷ compatibility with lattice gauge results in case of $\mu_B = \mu_S = \mu_Q = 0$.

Constructing "realistic" equation-of-state based on

- ▷ pressure p_H of a resonance gas,
- ▷ and pressure p_Q of an ideal quark gluon plasma, including bag pressure.

Usually $p = p_Q + \Theta(T_c - T)\,(p_H - p_Q)$, where p_H and p_Q cross at T_c.

The correct pressure

is assumed to be of the form

$$p = p_Q + \lambda\,(p_H - p_Q),$$

with

$$\lambda = \exp\left(-\frac{T - T_c}{\delta}\right)\Theta(T - T_c) + \Theta(T_c - T),$$

and

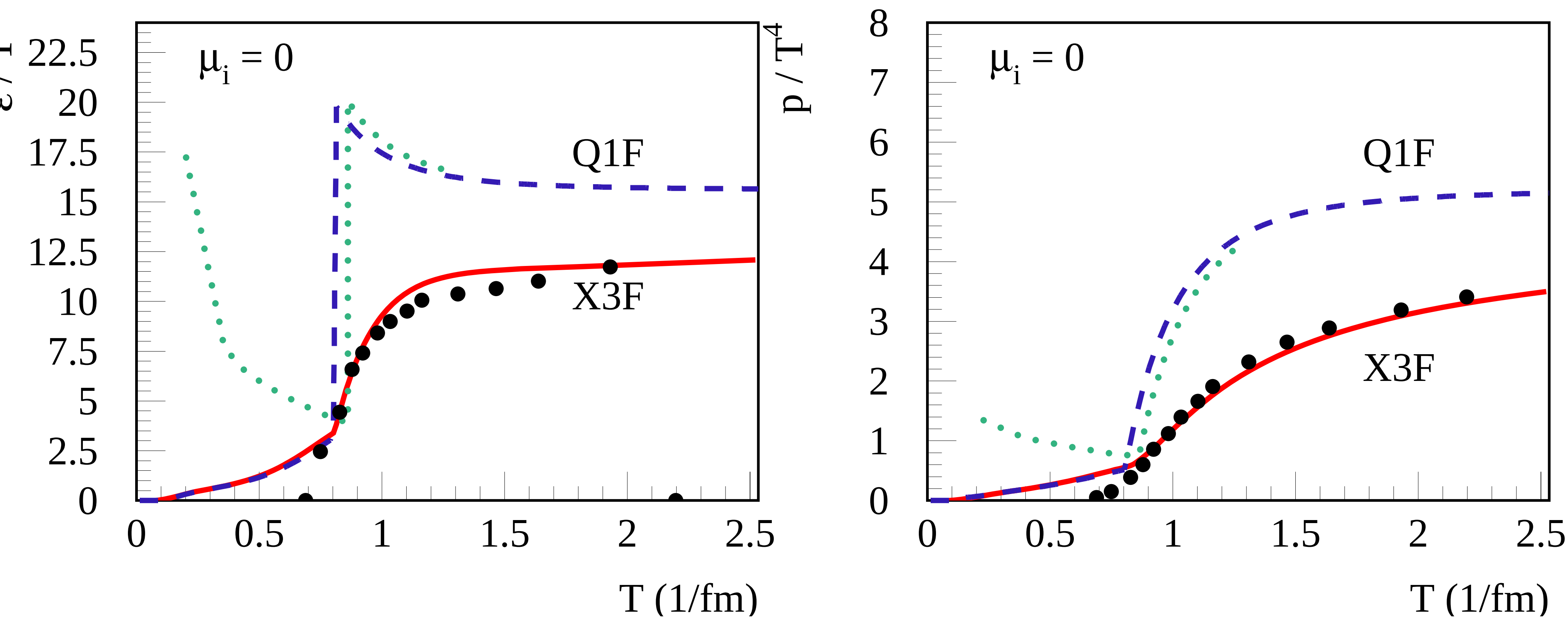
$$\delta = \delta_0 \exp\left(-(\mu_B/\mu_c)^2\right)\left(1 + \frac{T - T_c}{2T_c}\right).$$

From the pressure one obtains
the entropy density S, the flavor densities n^i, and the energy density as

$$S = \frac{\partial p}{\partial T},\; n^i = \frac{\partial p}{\partial \mu^i},\; \varepsilon = TS + \sum_i \mu^i n^i - p,$$

Our favorite equation-of-state ("X3F") is obtained for $\delta_0 = 0.15$,

which reproduces lattice gauge results for $\mu_B = \mu_S = \mu_Q = 0$:



The symbol X3F stands for "cross-over" and "3 flavor conservation"

Q1F: simple first order equation-of-state, with B conservation

dotted lines: EoS used by Hiano et al

At hadronization hypersurface, defined by a given temperature T_{H},
we switch from "matter" description to particles,
using the Cooper-Frye description

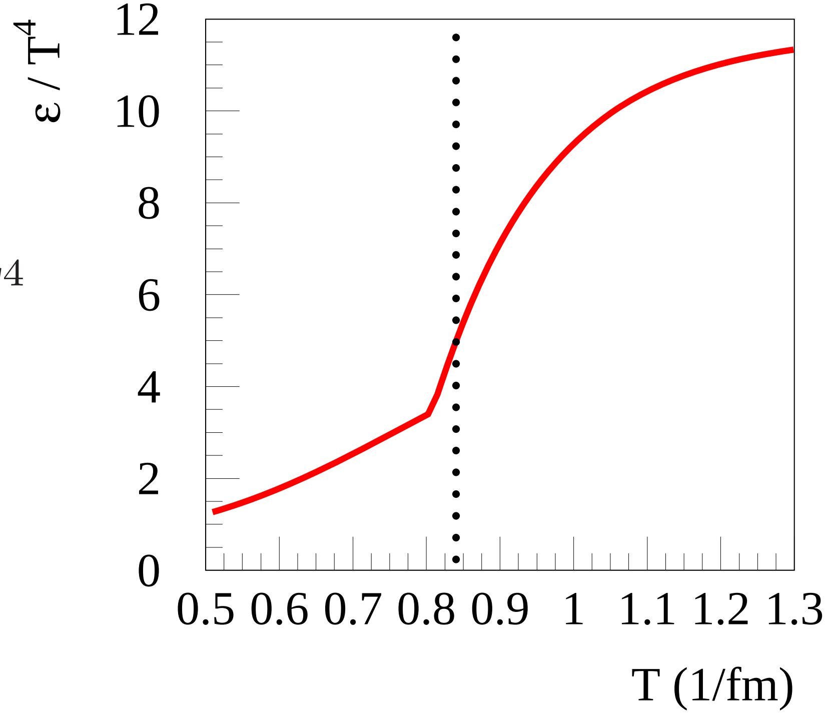
$$E\frac{dn}{d^3p} = d\Sigma_\mu p^\mu f(up),$$

Particles may still interact, so hadronization here means an intermediate stage, particles are not yet free streaming, but they are not thermalized any more.

After the hadronization, particles are fed into the hadronic cascade model UrQMD, performing hadronic interactions

Our favorite hadronization temperature T_H is 166 MeV (dotted line), which is right in the transition region!

Energy density over T^4 as a function of the temperature T

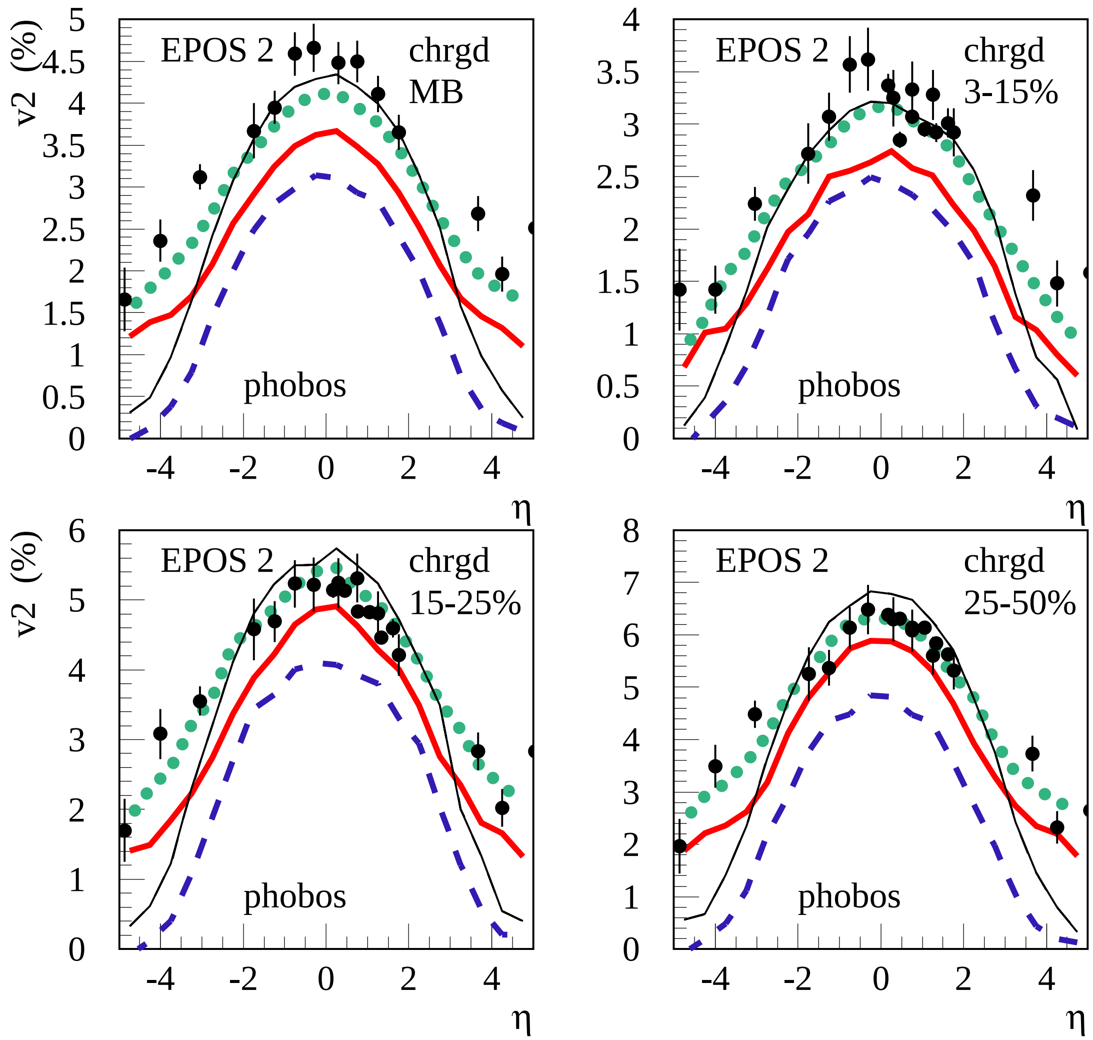


here: hadronization of the quark-gluon plasma state into a hadronic system, at an early stage, not the decay of a resonance gas in equilibrium!!

▷ After this hadronization – although no longer thermal– the system still interacts via hadronic scatterings,

▷ still building up (elliptical) flow, and therefore affecting slopes and azimuthal asymmetry observables,

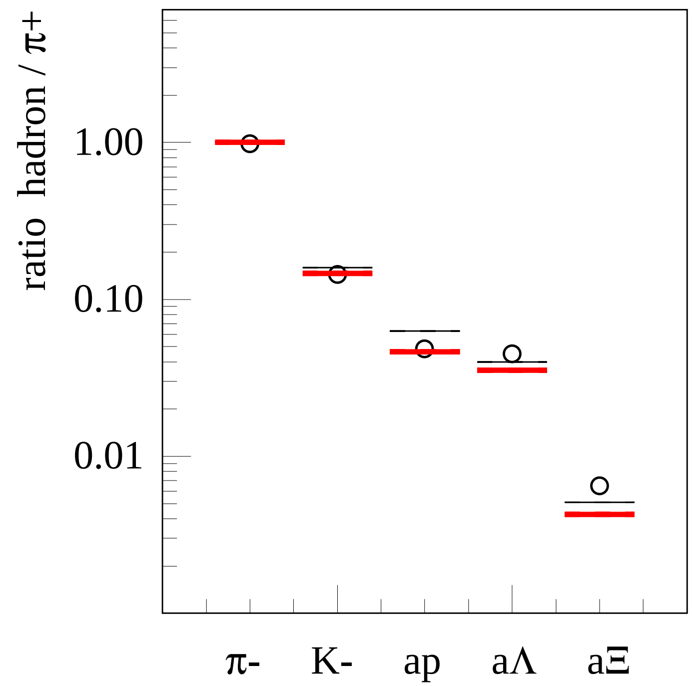
but less compared to an idealized thermal resonance gas evolution



However, despite the non-equilibrium behavior in the finale stage, we get particle yields close to what has been predicted in statistical models!
(final hadronic cascade does not change particle yields too much)

Particle ratios (hadron yields to π^+ yields)

from our model calculations (thick horizontal line) as compared to the statistical model (thin horizontal line), and to data (brahms, star) (points).

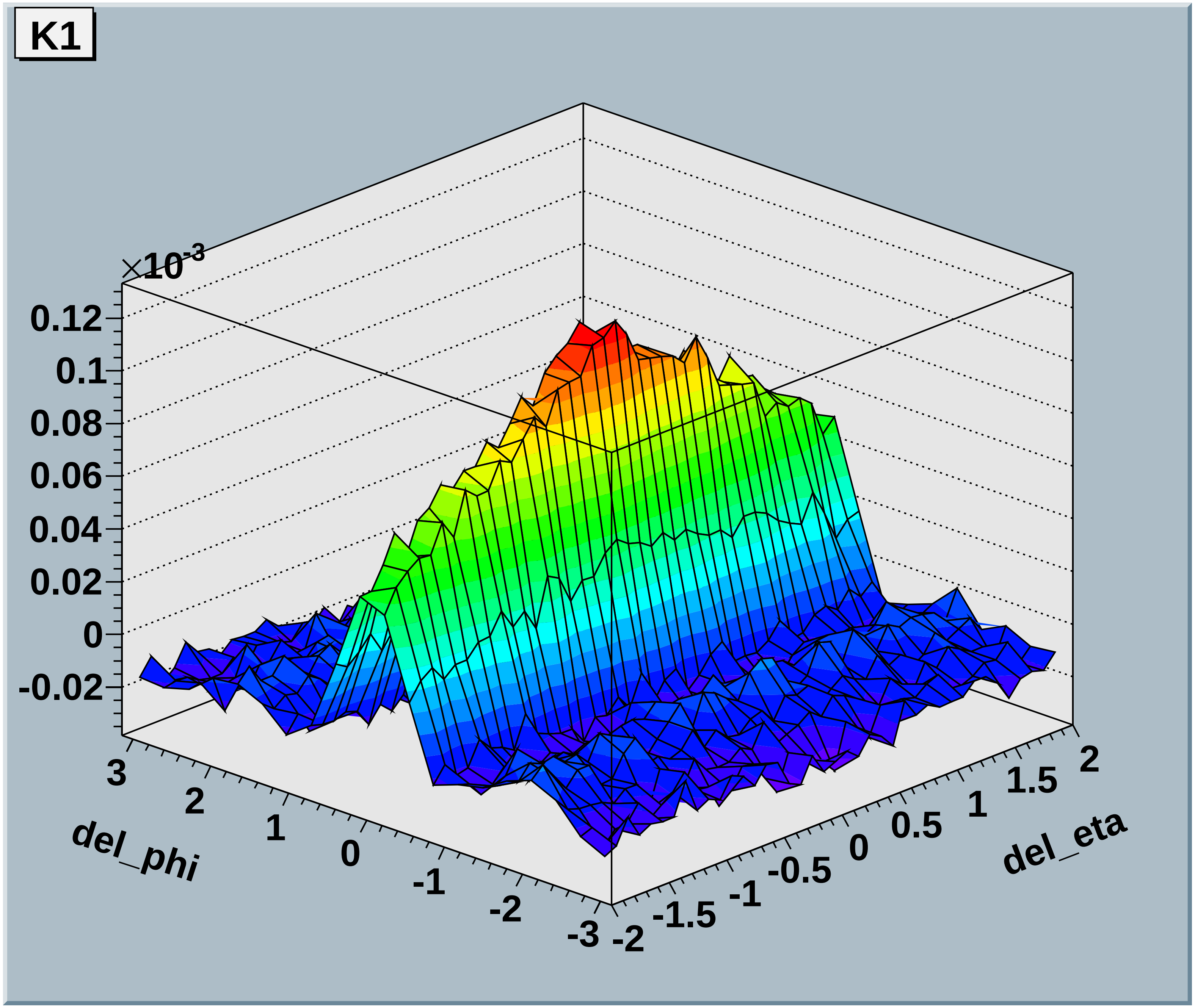


4 On the importance of an event-by-event treatment

Remarkable feature of an **event-by-event treatment** of the hydrodynamical evolution based on **random flux tube initial conditions**:

appearance of a so-called **ridge**-structure in the dihadron correlation $dN/d\Delta\eta\, d\Delta\phi$, with $\Delta\eta$ and $\Delta\phi$ being respectively the difference in pseudorapidity and azimuthal angle of a pair of particles.

Here, we consider trigger particles with transverse momenta between 3 and 4 GeV/c, and associated particles with transverse momenta between 2 GeV/c and the p_t of the trigger, in central Au-Au collisions at 200 GeV


K1
×10⁻³
0.12
0.1
0.08
0.06
0.04
0.02
0
-0.02
3
2
1
0
-1
-2
-3
del_phi
-2
-1.5
-1
-0.5
0
0.5
1
1.5
2
del_eta

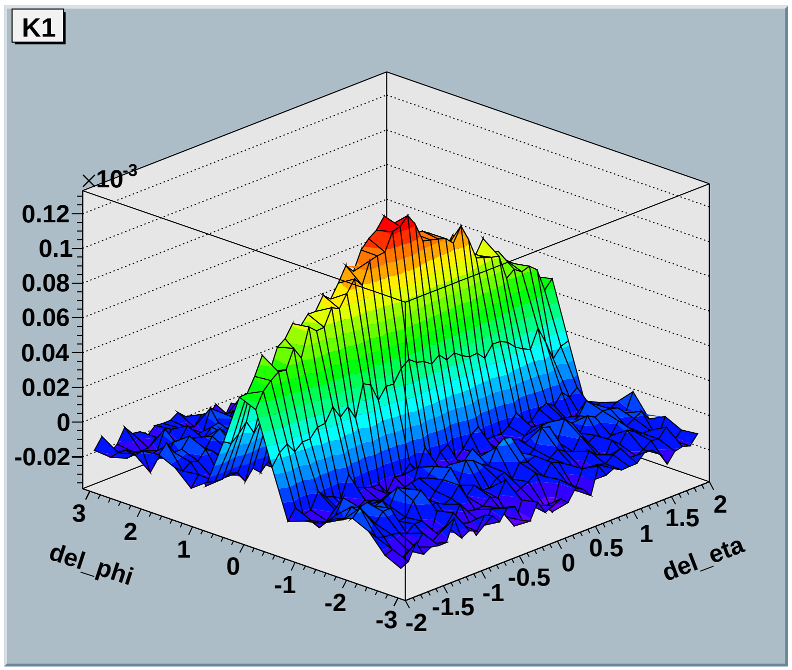
Spectacular result!

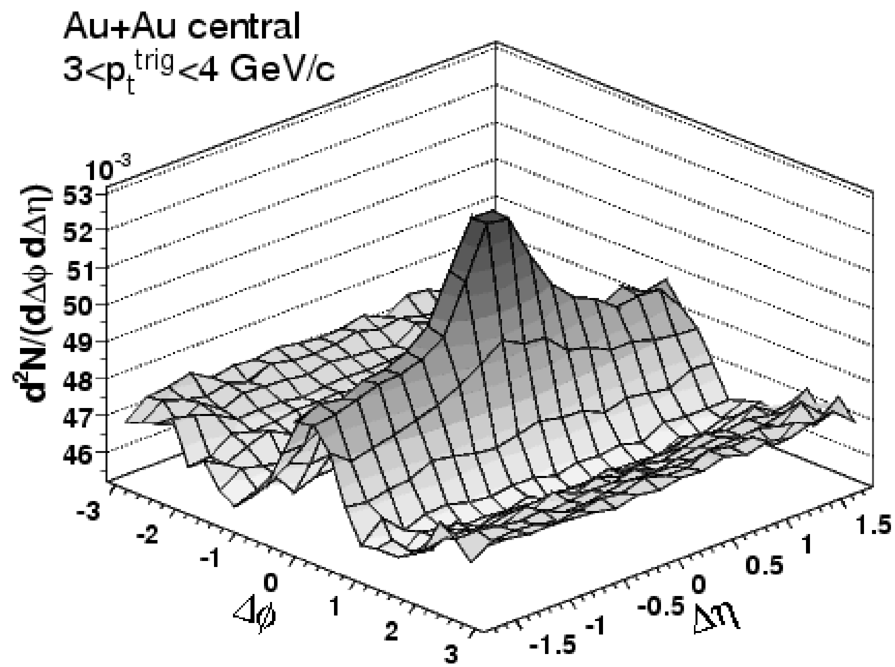
it means:

For a given particle emitted at some azimuthal angle ϕ at some pseudorapidity η, there is a large probability to observe at a quite different value of η a second particle emitted at the same angle!

EbE hydro

STAR data



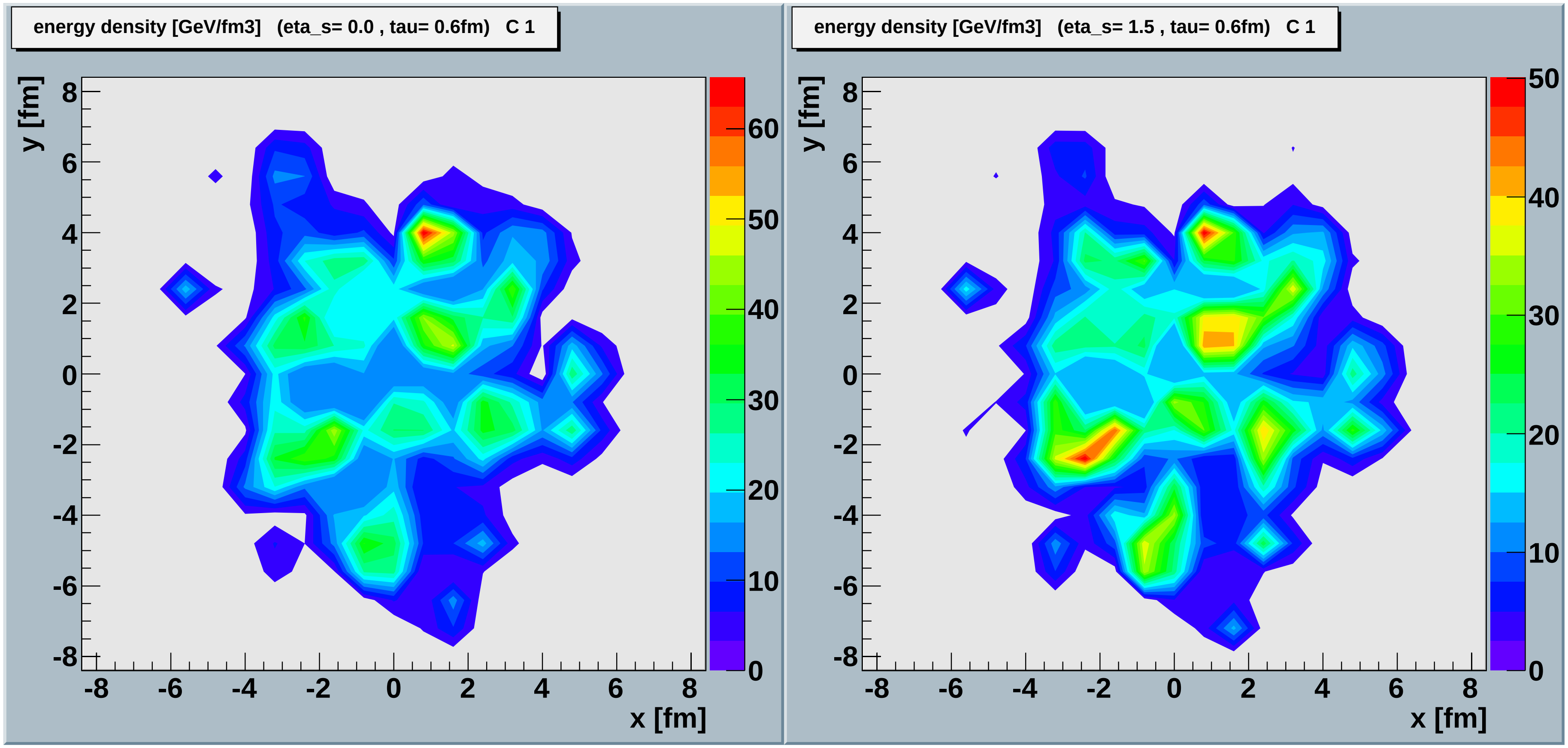


Similar ridge structure as in data
(somewhat less "jet" contribution due to soft/hard separation)

To understand

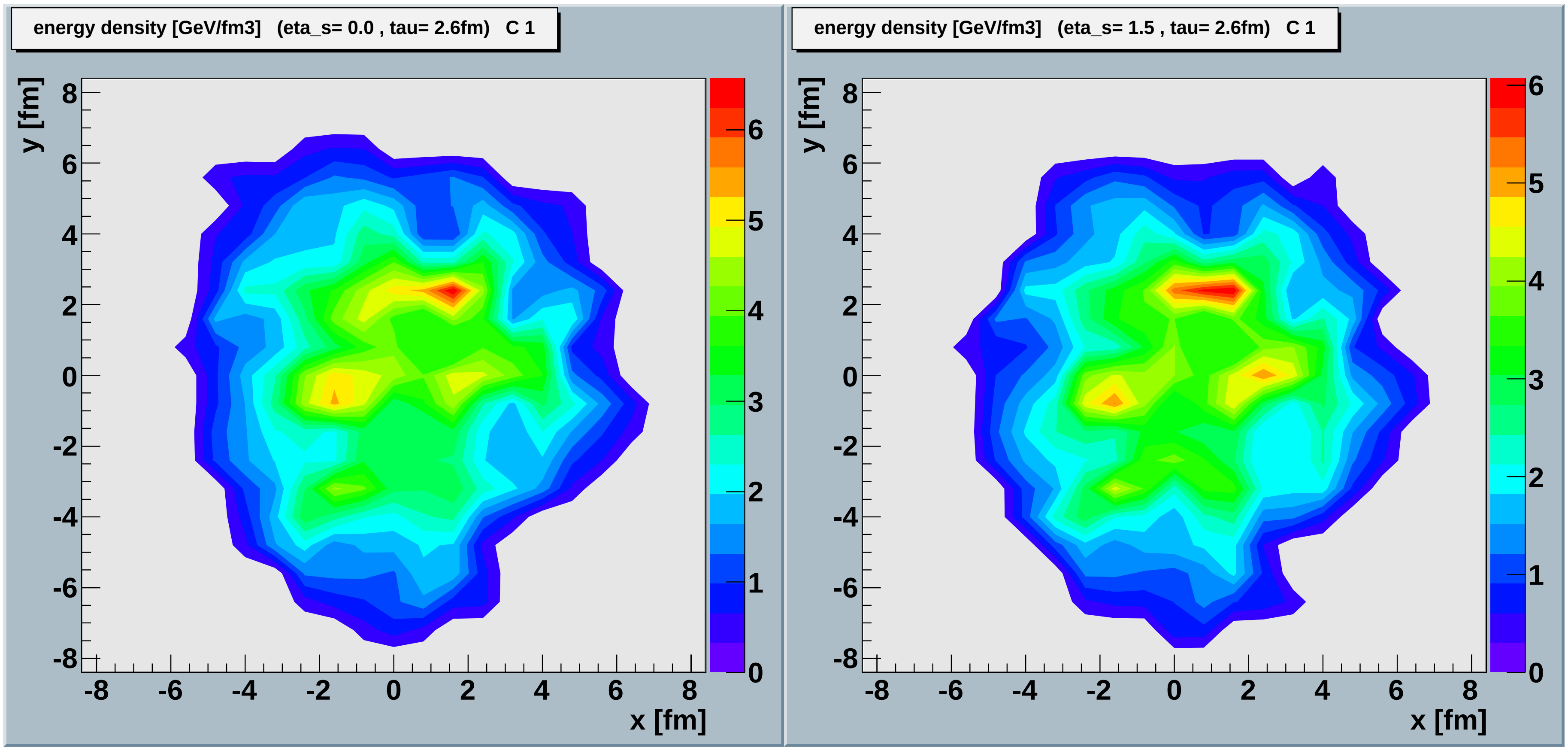
we consider in the following a single (but typical) event

Initial energy density in a central Au-Au collision at 200 GeV, at a space-time rapidity $\eta_s = 0$ and $\eta_s = 1.5$



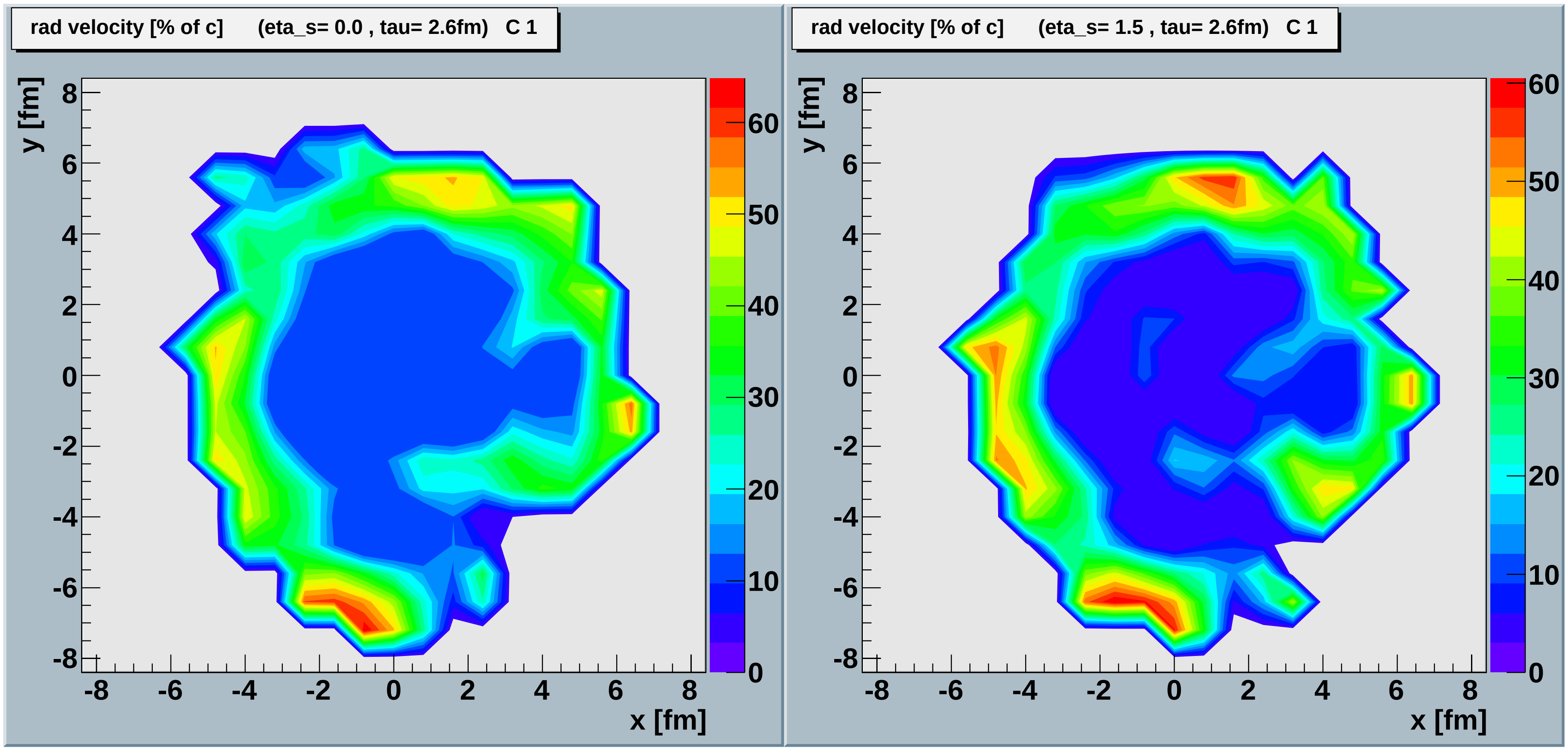
Sub-flux-tube structure (translational invariance)

Energy density at a proper time $\tau = 2.6\,\mathbf{fm/c}$,
at a space-time rapidity $\eta_s = 0$ and $\eta_s = 1.5$



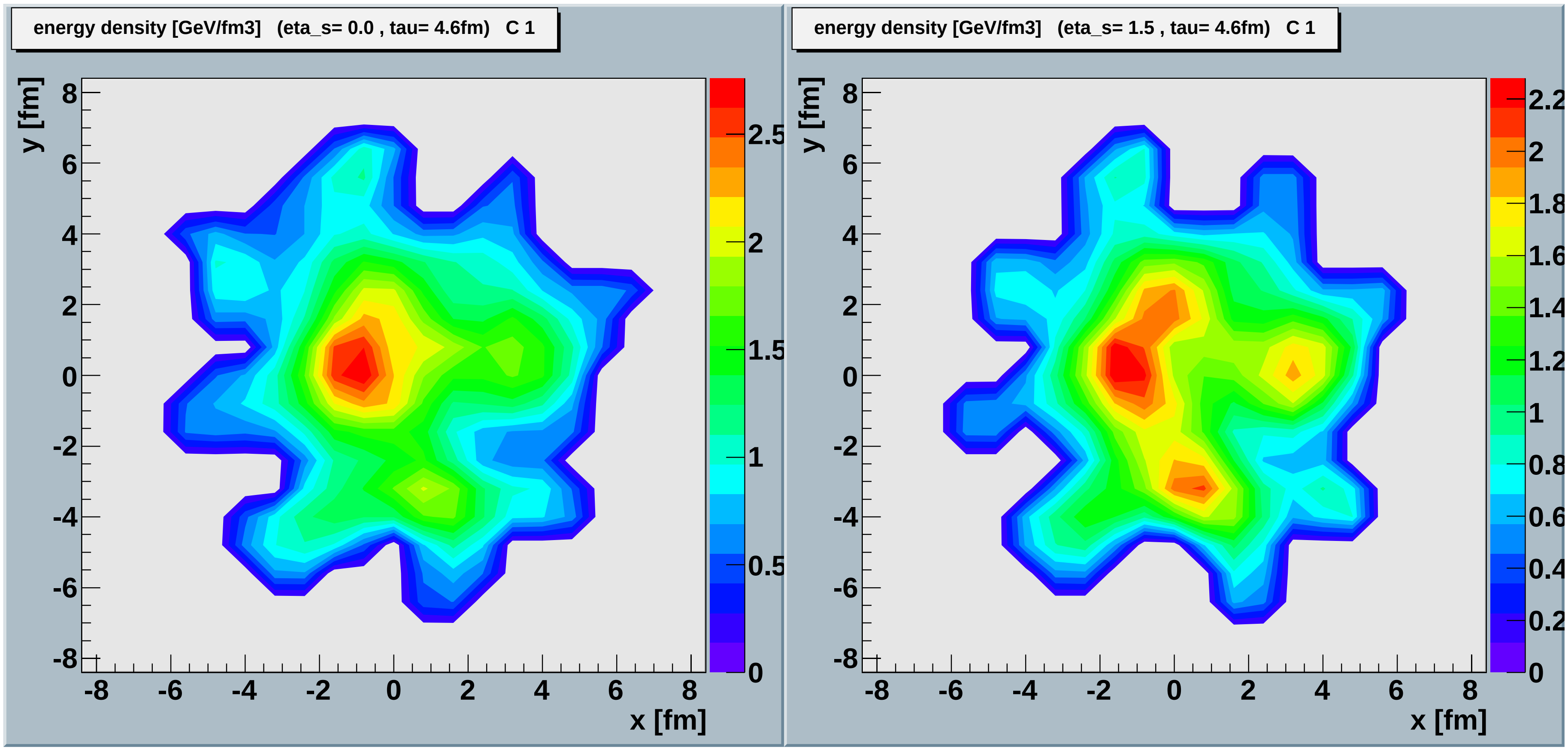
A bit later: still sub-flux-tube structure, but smoother

Radial flow velocity at a proper time $\tau = 2.6\,\mathbf{fm/c}$,
at a space-time rapidity $\eta_s = 0$ and $\eta_s = 1.5$



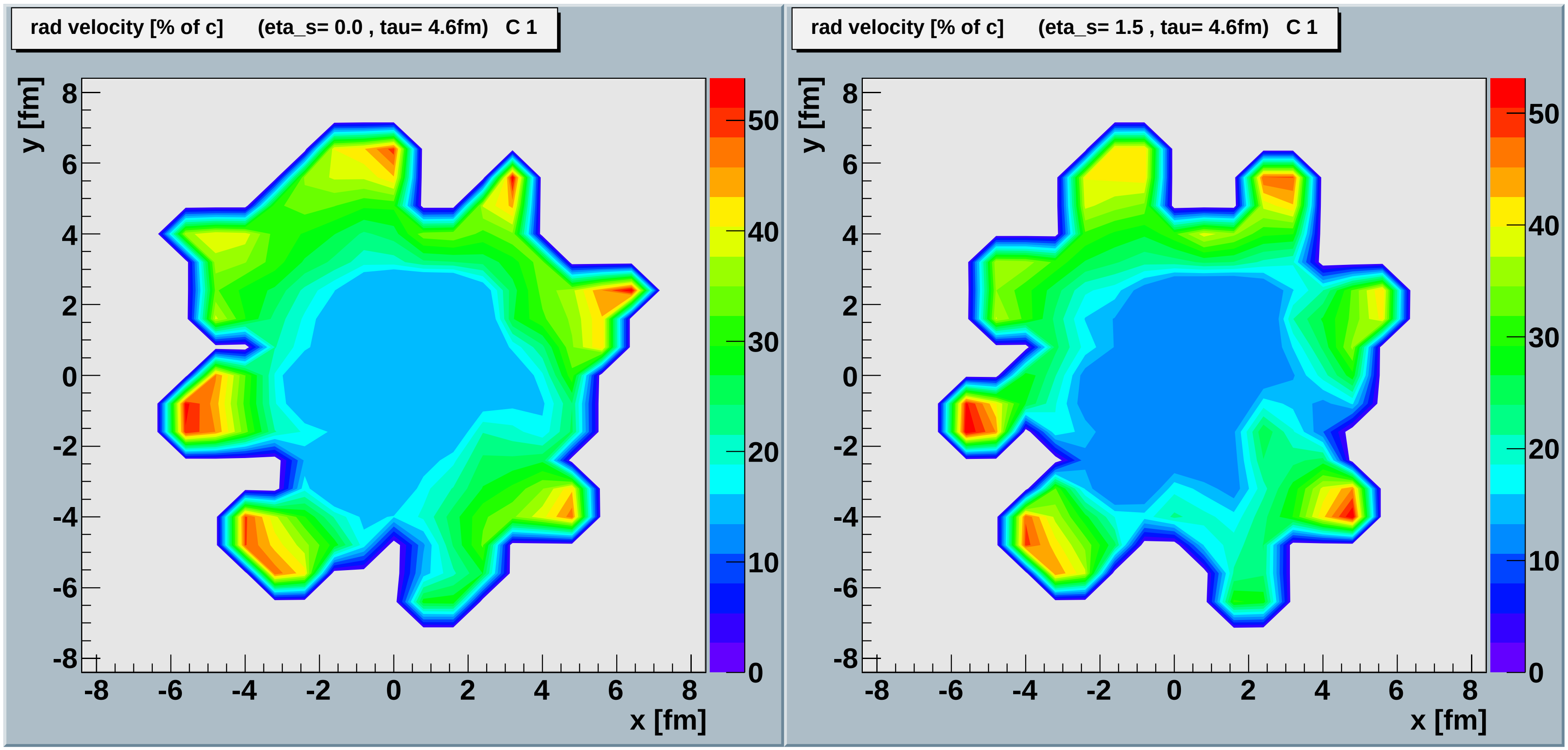
Flow: atoll-structure (peaks close to the border)

Energy density at a proper time $\tau = 4.6\,\mathbf{fm/c}$,
at a space-time rapidity $\eta_s = 0$ and $\eta_s = 1.5$.



Later due to the asymmetric flow: fingers

Radial flow velocity at a proper time $\tau = 4.6\,\mathbf{fm/c}$,
at a space-time rapidity $\eta_s = 0$ and $\eta_s = 1.5$.



with maximal flow in the fingers
and always translational invariance!!!

The well isolated peaks of the radial flow velocities have two important properties:

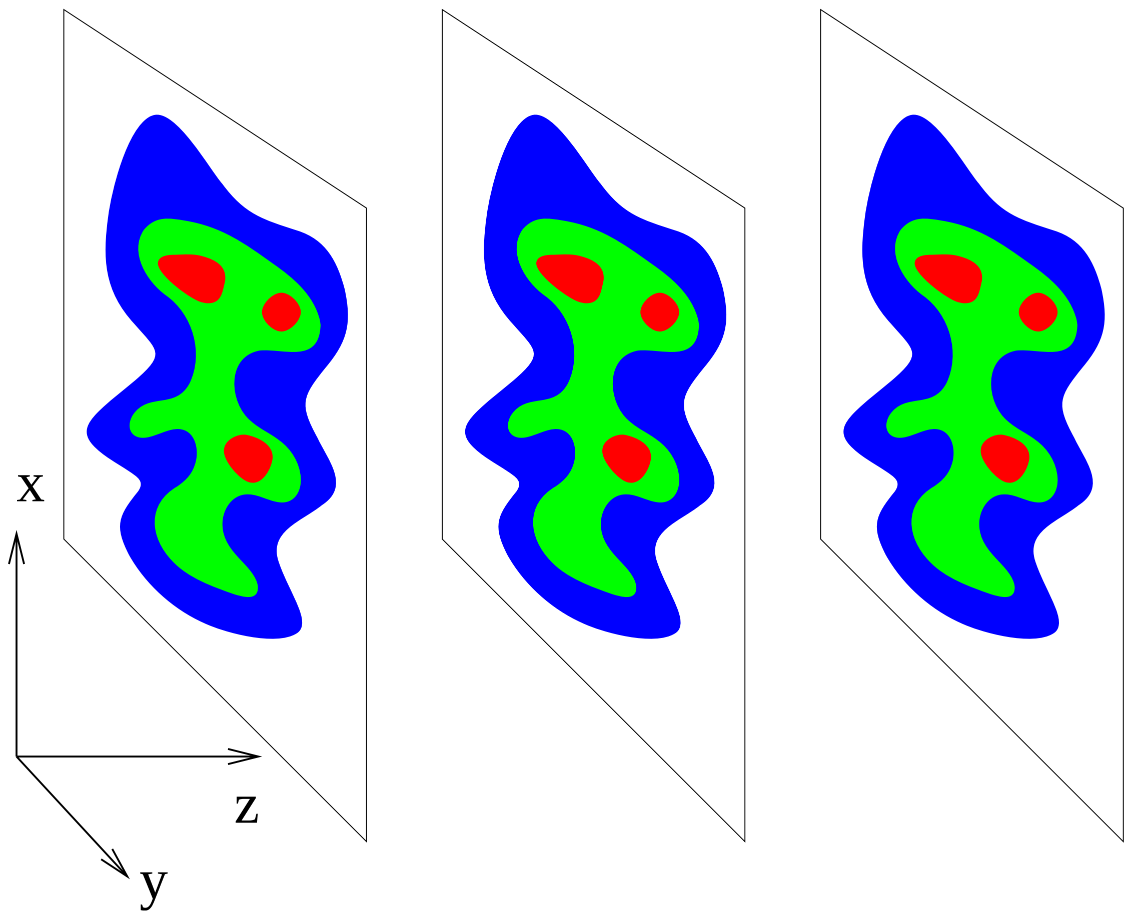
- ▷ they sit close to the hadronization surface,
- ▷ and they sit at the same azimuthal angle, when comparing different longitudinal positions η_s.

As a consequence, particles emitted from different longitudinal positions get the same transverse boost , when their emission points correspond to the azimuthal angle of a common flow peak position.

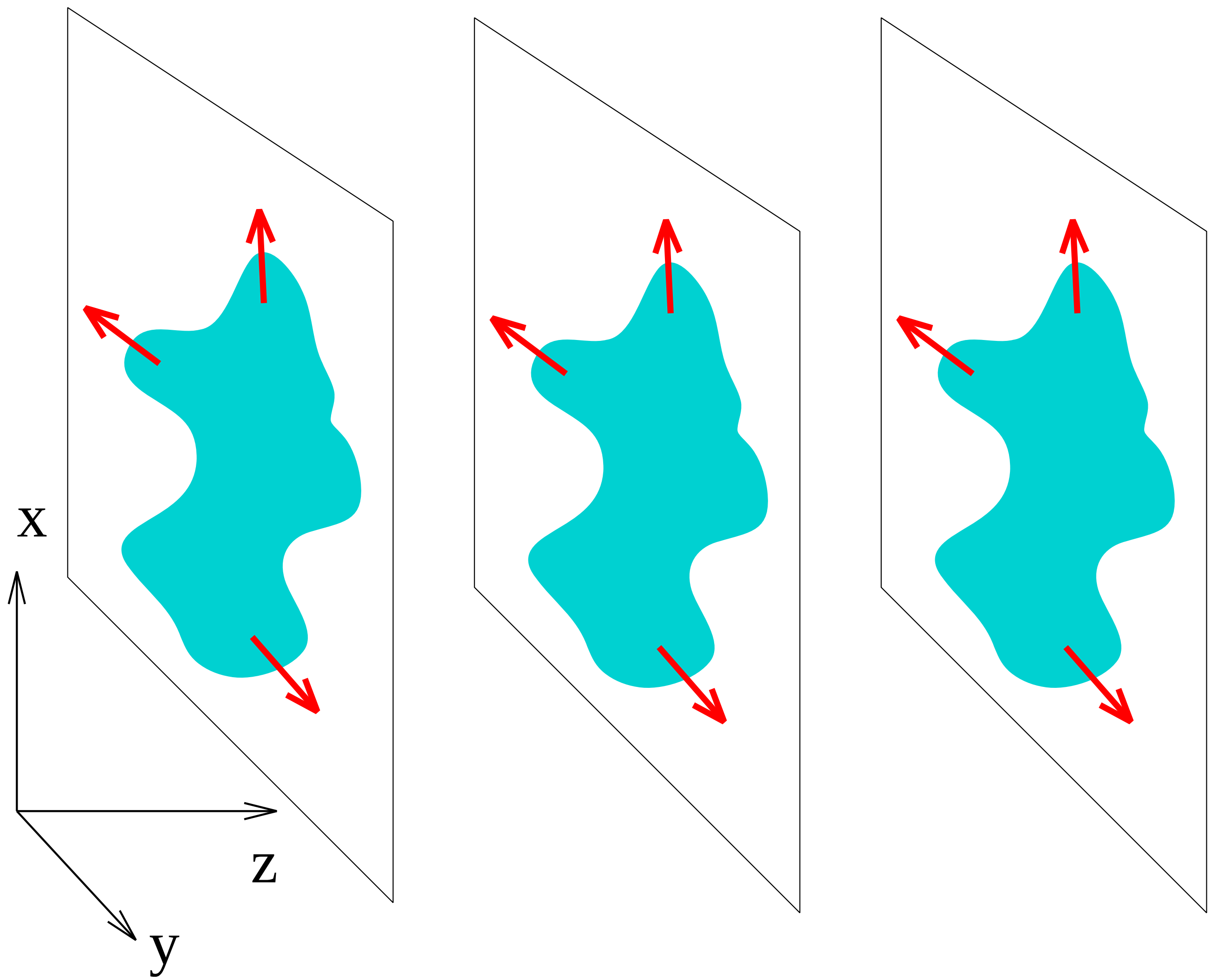
And since longitudinal coordinate and (pseudo)rapidity are correlated, one obtains finally a strong $\Delta\eta$ – $\Delta\phi$ correlation.

RIDGE summary

1) bumpy structure of energy density in transverse plane,
but translational invariance



2) this leads to translational invariance of transverse flows



3)

These identical flow patterns at different longitudinal positions

give the same collective push to particles produced at different values of η_s,

when they are emitted at an azimuthal angle corresponding to a flow maximum (indicated by the arrows in the figure).

Final question:

why do we have a irregular transverse structure with an approximate translational invariance?

The basic structure of EPOS is such that each individual nucleon-nucleon collision results in

- ▷ a projectile and target remnant,
- ▷ and two or more elementary flux tubes / strings. The higher the energy the bigger the number of strings.

The flux tubes cover only a limited range in rapidity,

but their "lengths" (in rapidity) vary enormously.

Nevertheless we obtain a very smooth variation of the energy density with the longitudinal coordinate η_s.

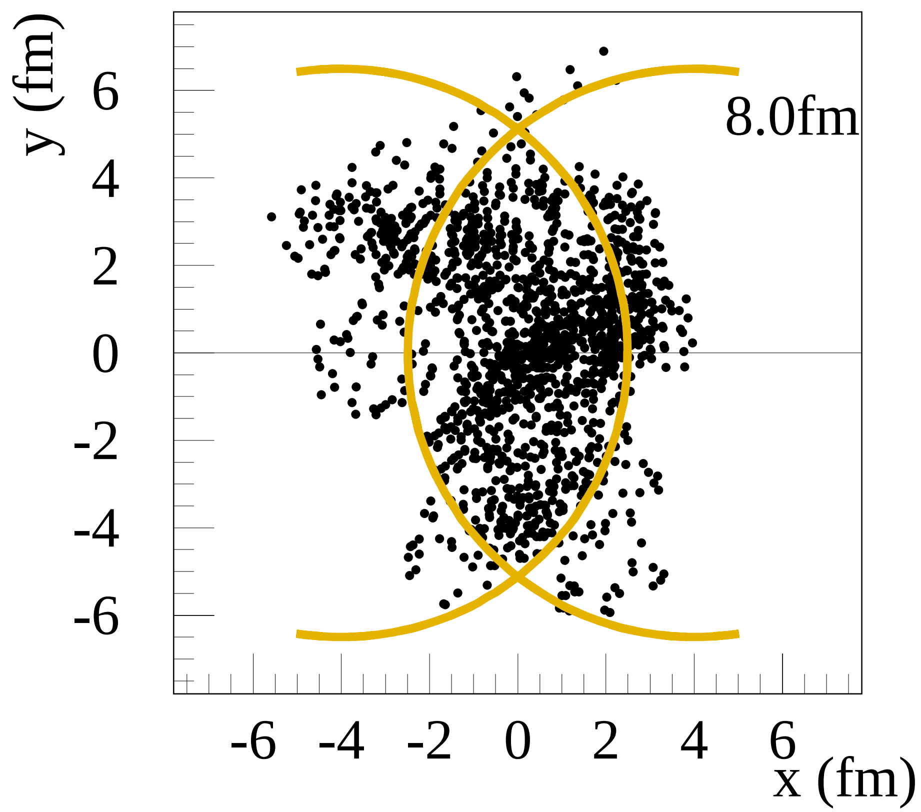
This is due to the fact that the transverse positions of a string is given by the position of the nucleon pair, who's interaction gave rise the the formation of the flux tube.

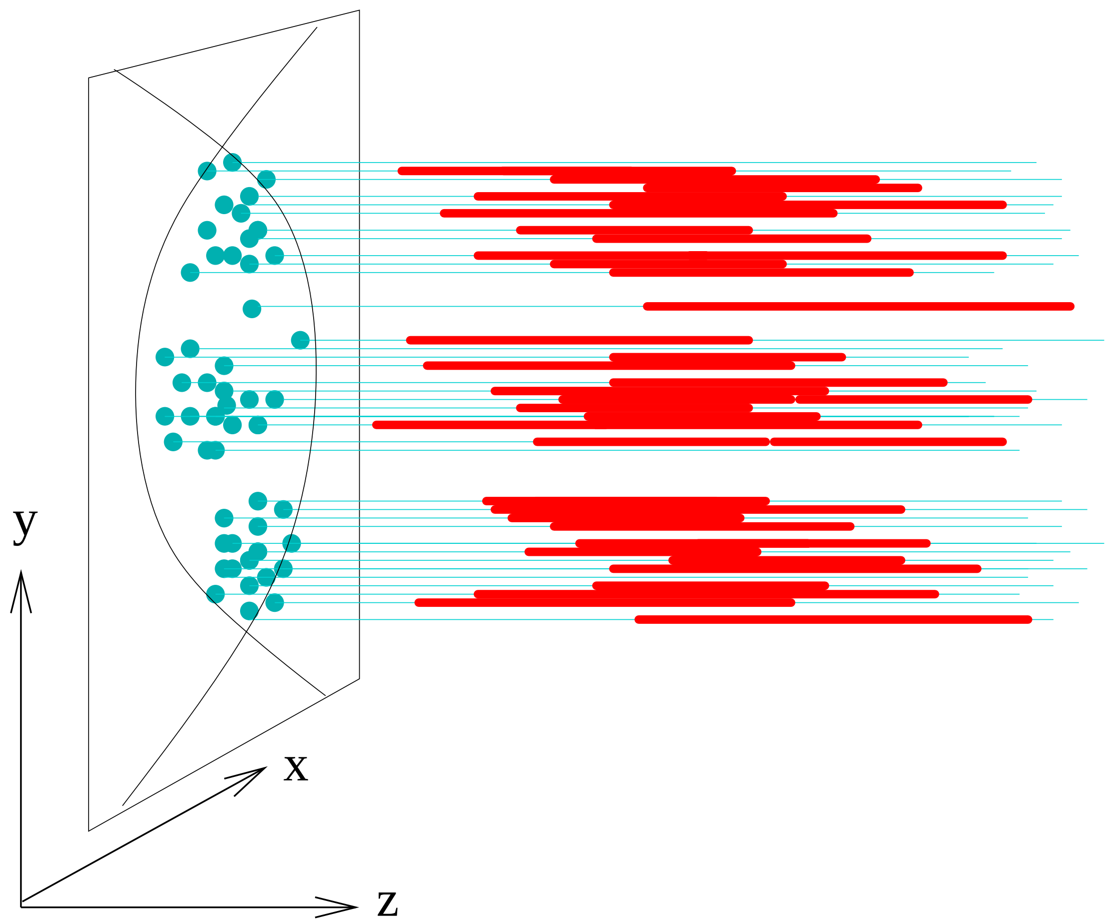
Semi-peripheral ($b = 8\text{fm/c}$) Au-Au event at 200 GeV:
projection of the positions of nucleon-nucleon scattering to the transverse (x, y) plane

inhomogeneous structure:

there are areas with a high density of interaction points, and areas which are less populated.

These transverse positions of interacting pairs define also the corresponding positions of the flux tubes associated to the pairs





Projection of the positions of nucleon-nucleon scattering to the transverse (x, y) plane, which defines "possible transverse positions" of the flux tubes (thin lines).

The actual flux tubes (thick red lines) fluctuate concerning their longitudinal positions (but nevertheless add up to sub-flux-tubes)

So:

Sub-flux-tubes in EPOS (translational invariance),
traced back to NN scattering positions

lead to irregular (asymmetric) translational invariant flows

which lead to the "ridge" effect

Phantastic:

one has experimantally access to non-trivial flow structures of sizes of 1 fermi !!!

more details:

http://arxiv.org/abs/1004.0805