Kinematic Fitting

Armen Kechechyan

JINR, Dubna

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Nantes

Outline

- ✓ Introduction
- ✓ Algorithm
- ✓ Testing
- ✓ STAR analysis
- ✓ Summary

Introduction

Kinematic fitting is a mathematical procedure in which one uses the physical laws governing a particle interaction or decay ***to improve the measurements of the process, test hypothesis and to find unknown parameters.***

Physical information is supplied via ***constraints***. Each constraint is expressed in the form of an equation manifesting some physical condition that the process must satisfy.

Constraints are generally implemented through a ***least-squares procedure***. Each constraint equation is added via the ***Lagrange multiplier technique*** to the χ^2 equation of the tracks using the covariance matrices of the tracks.

General algorithm

The expression for χ^2 and constraints can be written in the matrix form:

$$\chi^2 = (\boldsymbol{\alpha} - \boldsymbol{\alpha}_0)^T \mathbf{V}_{\alpha_0}^{-1} (\boldsymbol{\alpha} - \boldsymbol{\alpha}_0)$$

$$\mathbf{H}(\boldsymbol{\alpha}) = \mathbf{0} \quad - \text{constraints}$$

parameters for a set of tracks

$$\alpha \equiv \begin{pmatrix} \alpha_1 \\ \vdots \\ \alpha_n \end{pmatrix}$$

measurement parameters

$$\alpha_0 \equiv \begin{pmatrix} \alpha_{01} \\ \vdots \\ \alpha_{0n} \end{pmatrix}$$

the inverse measurement covariant matrix

$$V_{\alpha_0}^{-1} \equiv \begin{pmatrix} 1/\sigma_1^2 & \dots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \dots & 1/\sigma_n^2 \end{pmatrix}$$

Vector of r constraints (nonlinear equality)

$$\mathbf{H}(\boldsymbol{\alpha}) \equiv \left(H_1(\boldsymbol{\alpha}), \dots, H_r(\boldsymbol{\alpha}) \right)$$

General algorithm

In the mathematics this problem known as ***nonlinear minimization problem with nonlinear equality constraints***.

Simplification: *Linearization of the equation – expanding around a convenient point* $\boldsymbol{\alpha}_A$

$$\mathbf{H}(\boldsymbol{\alpha}) = \mathbf{0}$$

$$[\mathbf{H}(\boldsymbol{\alpha}_A) + \partial\mathbf{H}(\boldsymbol{\alpha}_A)/\partial\boldsymbol{\alpha}](\boldsymbol{\alpha} - \boldsymbol{\alpha}_A) \equiv \mathbf{D}\delta\boldsymbol{\alpha} + \mathbf{d} = \mathbf{0}$$

$$\mathbf{D} \equiv \begin{pmatrix} \frac{\partial H_1}{\partial \alpha_1} & \cdots & \frac{\partial H_1}{\partial \alpha_n} \\ \vdots & \ddots & \vdots \\ \frac{\partial H_n}{\partial \alpha_1} & \cdots & \frac{\partial H_n}{\partial \alpha_n} \end{pmatrix}$$

$$\delta\boldsymbol{\alpha} = \boldsymbol{\alpha} - \boldsymbol{\alpha}_A$$

$$\mathbf{d} \equiv \begin{pmatrix} H_1(\boldsymbol{\alpha}_A) \\ \vdots \\ H_r(\boldsymbol{\alpha}_A) \end{pmatrix}$$

General algorithm

The constraints are incorporated by using the method of Lagrange multipliers in which the χ^2 is written as a sum of two terms:

$$\chi^2 = \left(\boldsymbol{\alpha} - \boldsymbol{\alpha_0}\right)^T \mathbf{V}_{\boldsymbol{\alpha_0}}^{-1} \left(\boldsymbol{\alpha} - \boldsymbol{\alpha_0}\right) + 2\boldsymbol{\lambda}^T (\mathbf{D}\delta\boldsymbol{\alpha} + \mathbf{d})$$

$\boldsymbol{\lambda}$ is a vector of Lagrange multipliers

Minimizing the χ^2 with respect to $\boldsymbol{\alpha}$ and $\boldsymbol{\lambda}$ yields two vector equations

$$\mathbf{V}_{\boldsymbol{\alpha}_0}^{-1} \left(\boldsymbol{\alpha} - \boldsymbol{\alpha}_0\right) = \mathbf{0}$$

$$\mathbf{D}\delta\boldsymbol{\alpha} + \mathbf{d} = \mathbf{0}$$

which can be solved for the parameters and their covariance matrix.

General algorithm

The solution of the system of vector equations

$$\boldsymbol{\alpha} = \boldsymbol{\alpha}_0 - \mathbf{V}_{\alpha_0}\mathbf{D}^T\boldsymbol{\lambda}$$

$$\boldsymbol{\lambda} = \mathbf{V}_D\left(\mathbf{D}\delta\boldsymbol{\alpha}_0 + \mathbf{d}\right)$$

$$\mathbf{V}_D = \left(\mathbf{D}\mathbf{V}_{\alpha_0}\mathbf{D}^T\right)^{-1}$$

$$\delta\boldsymbol{\alpha}_0 = \boldsymbol{\alpha}_0 - \boldsymbol{\alpha}_A$$

New χ^2 and covariant matrix

$$\mathbf{V}_\alpha = \mathbf{V}_{\alpha_0} - \mathbf{V}_{\alpha_0}\mathbf{D}^T\mathbf{V}_D\mathbf{D}\mathbf{V}_{\alpha_0}$$

$$\chi^2 = \boldsymbol{\lambda}^T\mathbf{V}_D^{-1}\boldsymbol{\lambda} = \boldsymbol{\lambda}^T\left(\mathbf{D}\delta\boldsymbol{\alpha}_0 + \mathbf{d}\right)$$

Iterative procedure:

1 step: $\alpha_A = \alpha_0$ first approximation $\alpha^{(0)}$

2 step: $\alpha_A = \alpha^{(0)}$ second approximation $\alpha^{(1)}$

…

Convergence criterion

$$|(\chi^2)^{i+1} - (\chi^2)^i| < \varepsilon$$

Test of the fitting algorithm

The ***kinematic fitting*** procedure was applied for the analysis of the K_S^0 decay

$$K_S^0 \rightarrow \pi^+ + \pi^-$$

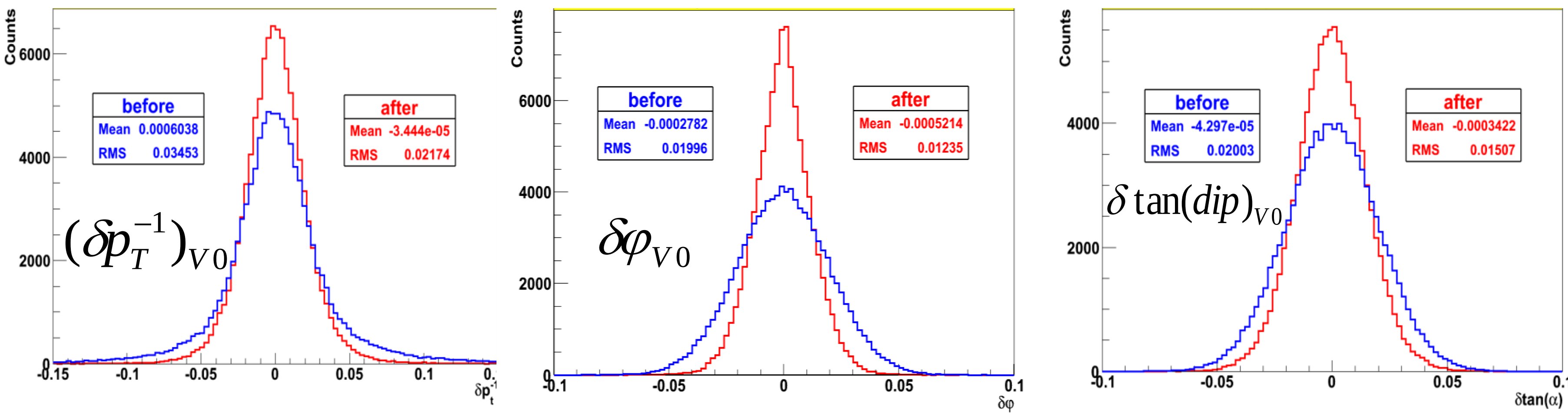
The set of the fitting parameters for each particle

$$\alpha = (1/p_T, \varphi, \tan(dip))$$

Constraints (4-momentum conservation law)

$$p^{\mu}_{K_s^0} - p^{\mu}_{\pi^+} - p^{\mu}_{\pi^-} = 0, \qquad \mu = 1,2,3,4$$

Toy Monte Carlo simulation with Gaussian shape errors



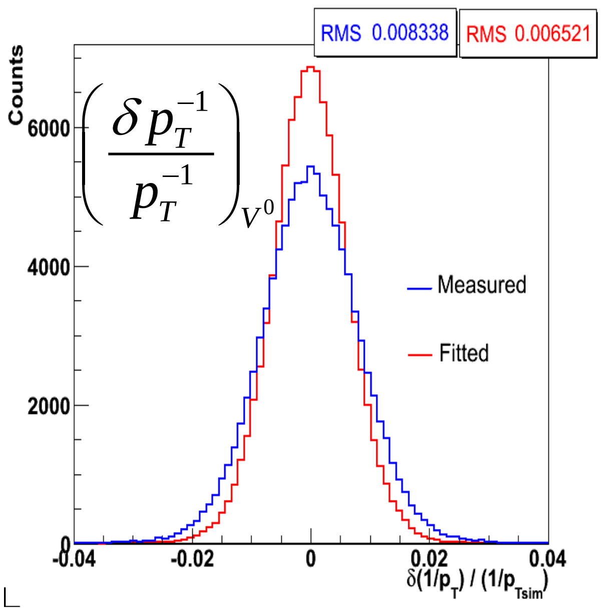
The resolution of "experimental" measurements improved

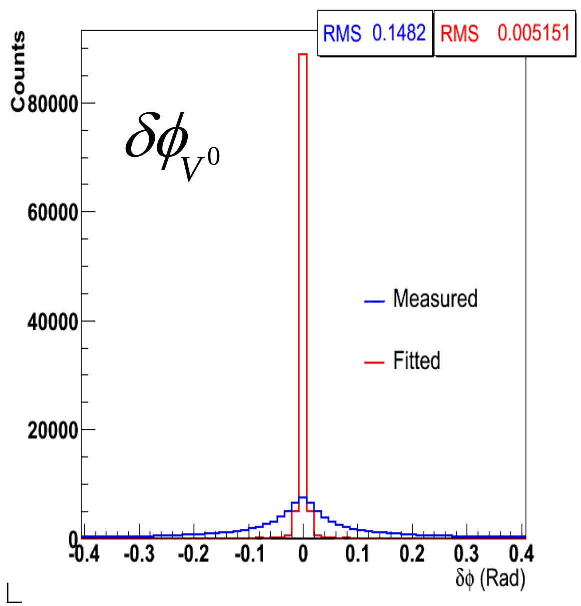
Test of kinematic & geometrical fitting

The set of the fitting parameters for each particle

$$\alpha = (1/p_T, \phi, \tan(dip), x, y, z)$$

Toy Monte Carlo simulation with Gaussian shape errors





$$\delta Par = Par_{meas(fit)} - Par_{sim},$$

$$Par \equiv (1/p_T)_{V^0}, \phi_{V^0}, \tan(dip_{V^0}), x_{V^0}, y_{V^0}, z_{V^0}$$

Kinematical constraints

$$p^{\mu}_{K_s^0} - p^{\mu}_{\pi^+} - p^{\mu}_{\pi^-} = 0, \qquad \mu = 1,2,3,4$$

Geometrical constraints (helices intersection)

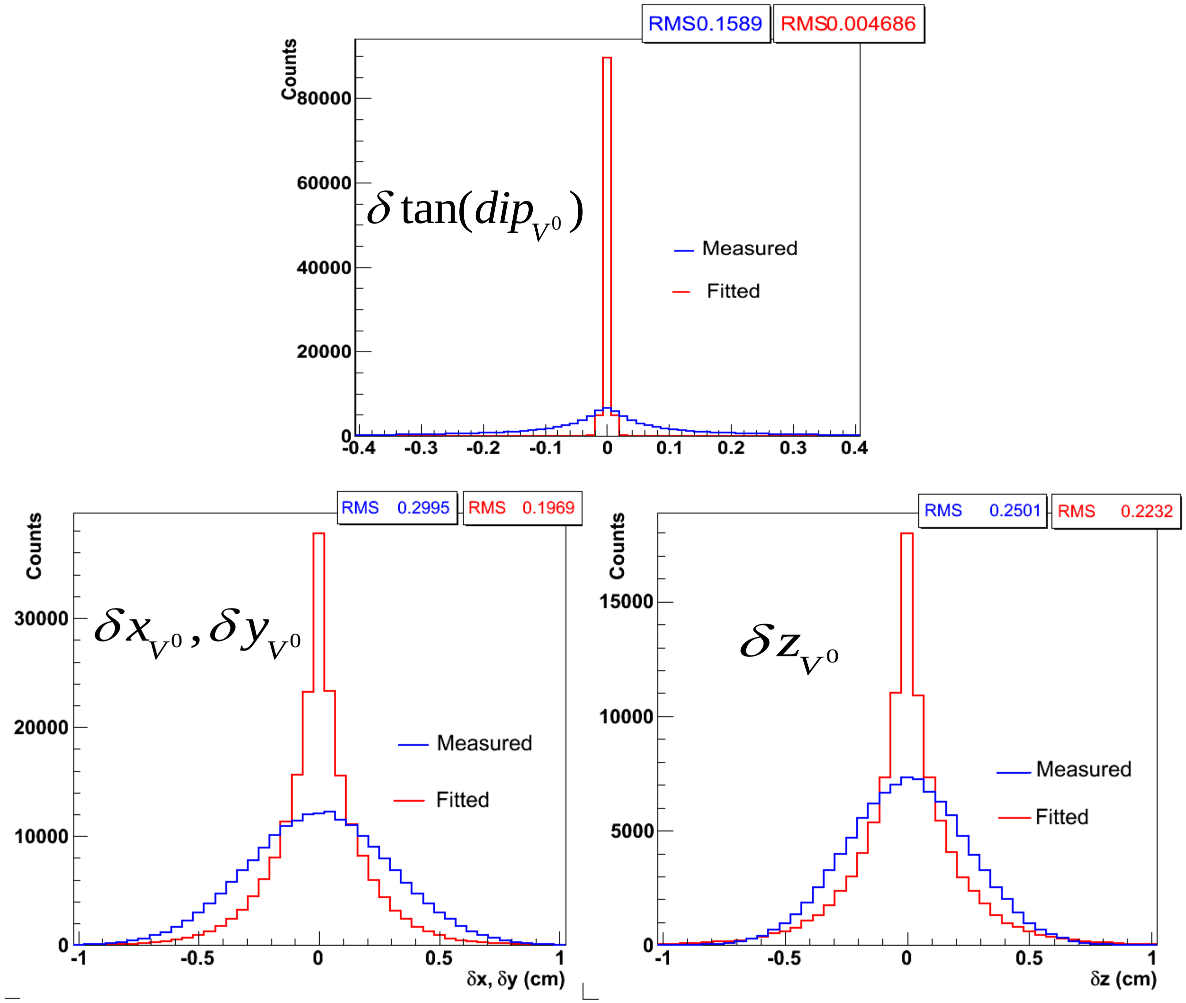
$$\vec{r}_{\pi^+}(s_1) - \vec{r}_{\pi^-}(s_2) = 0$$

$$\vec{r}(s) \equiv \left(x(s), y(s), z(s)\right)$$

Parameters	σ
$1/p_T$	1%
φ	0.005 Rad
tan(dip)	0.005
x_0 , y_0	0.085 cm
z_0	0.125 cm
x_{V0}, y_{V0}	0.3 cm
z_{V0}	0.25 cm

The resolution of "experimental" measurements improved

Test of kinematic & geometrical fitting (con'd)

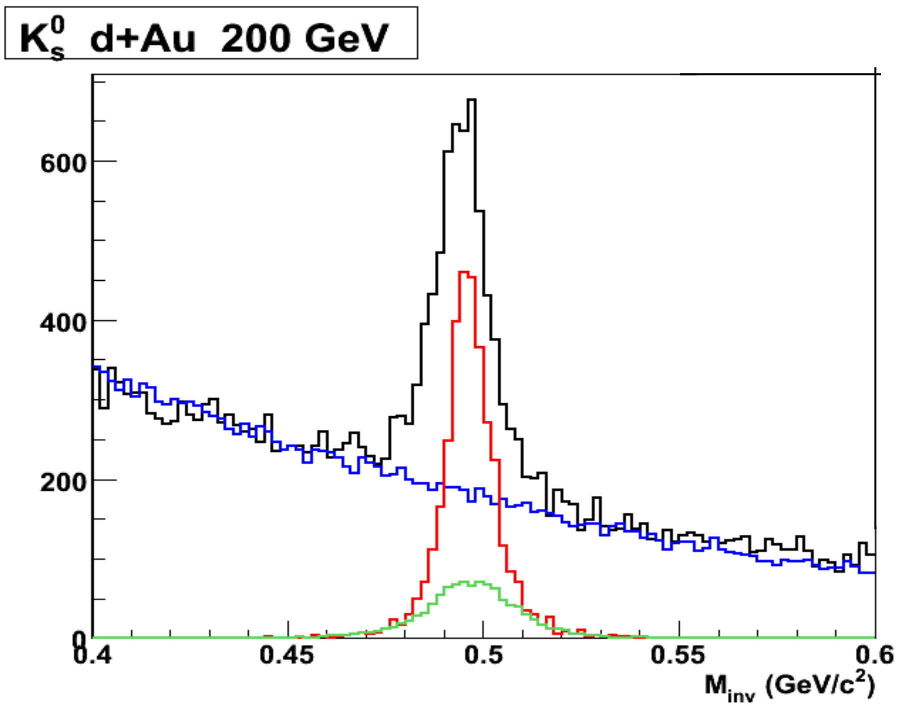


V0 vertex space resolution is improved

STAR Simulation & dAu@200 GeV

$$K_S^0 \rightarrow \pi^+ + \pi^-$$

HIJING-3.82 + GEANT + Reconstruction ***175K MinBais events***



— topological cuts

— background (mix)

— kinematic fit

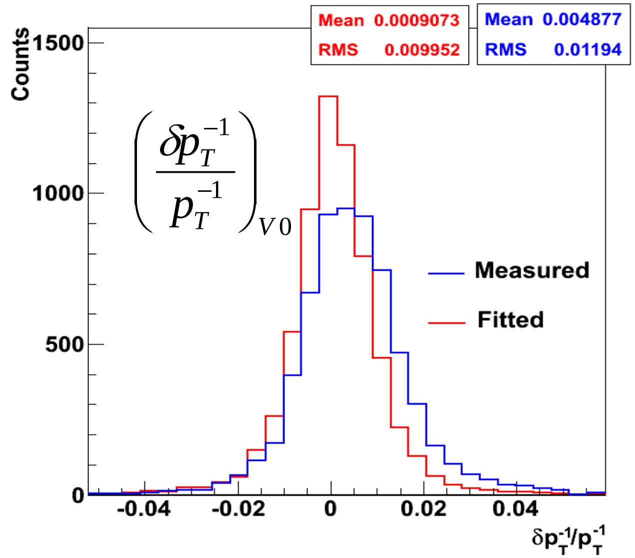
— background V^0's after fit

Signal / Background $\approx$ 3.4

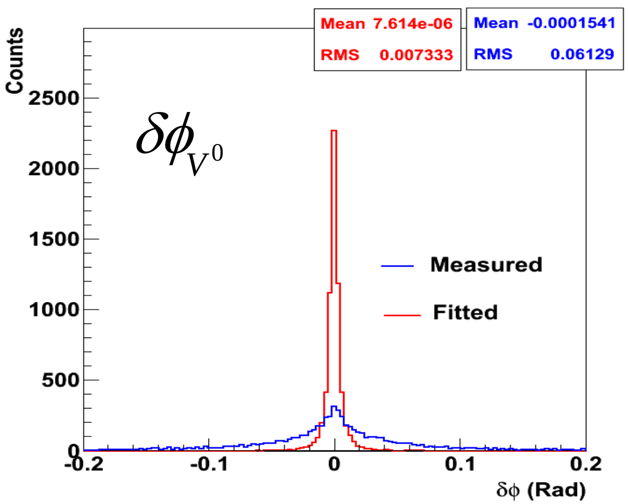
Strong suppression of background

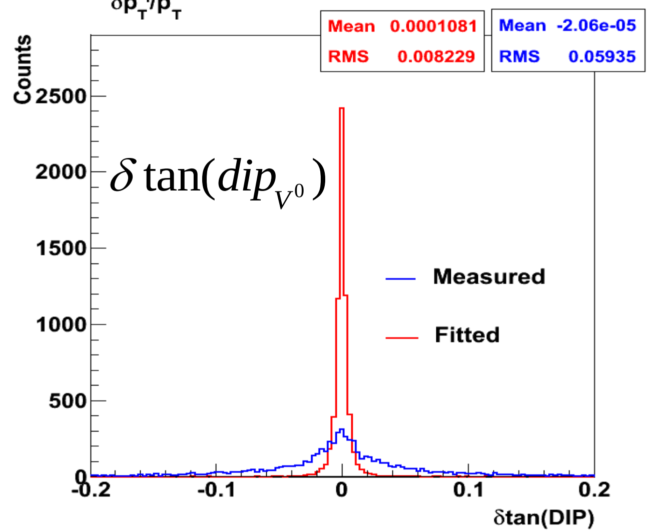
STAR Simulation & dAu@200 GeV (con'd)

$$K_S^0 \rightarrow \pi^+ + \pi^-$$



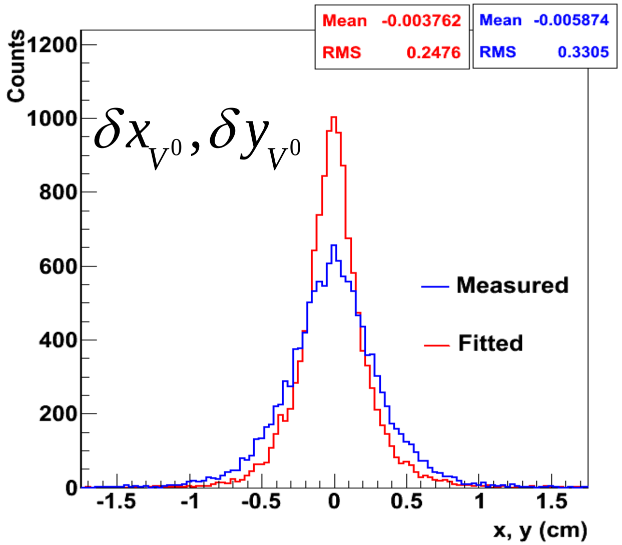
The mean value is corrected

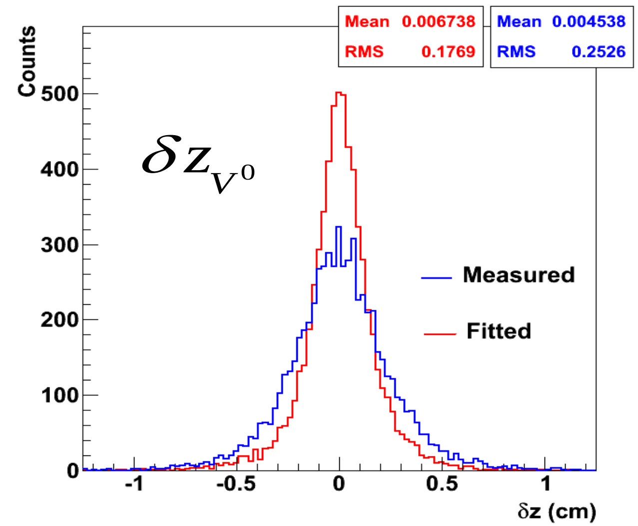




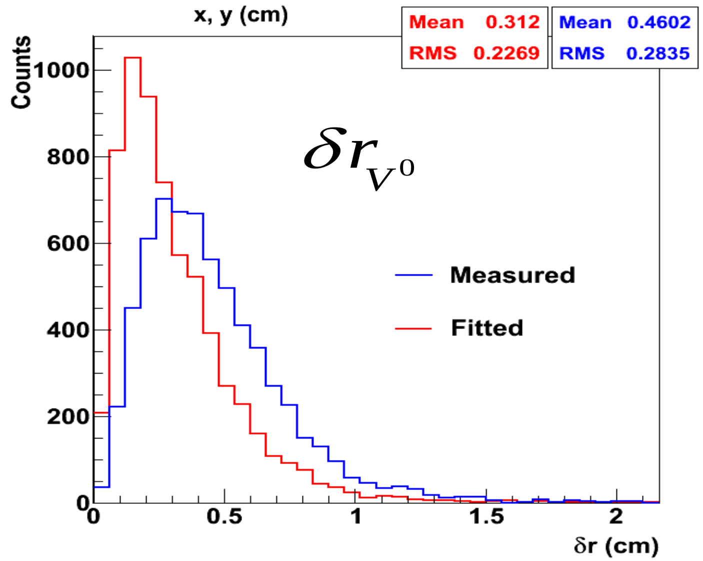
The resolution of "experimental" measurements are improved

STAR Simulation & dAu@200 GeV (con'd)





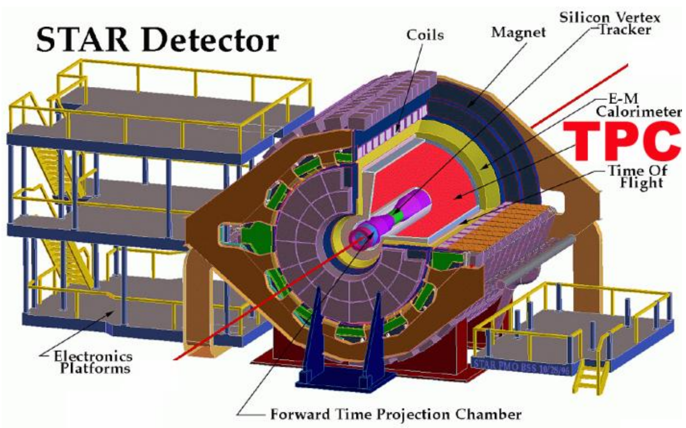
$$K_S^0 \rightarrow \pi^+ + \pi^-$$



$$\delta r_{V^0} = |\vec{r}_{\exp/\,fit V^0} - \vec{r}_{MC V^0}|$$

V^0 vertex space resolution is significantly improved

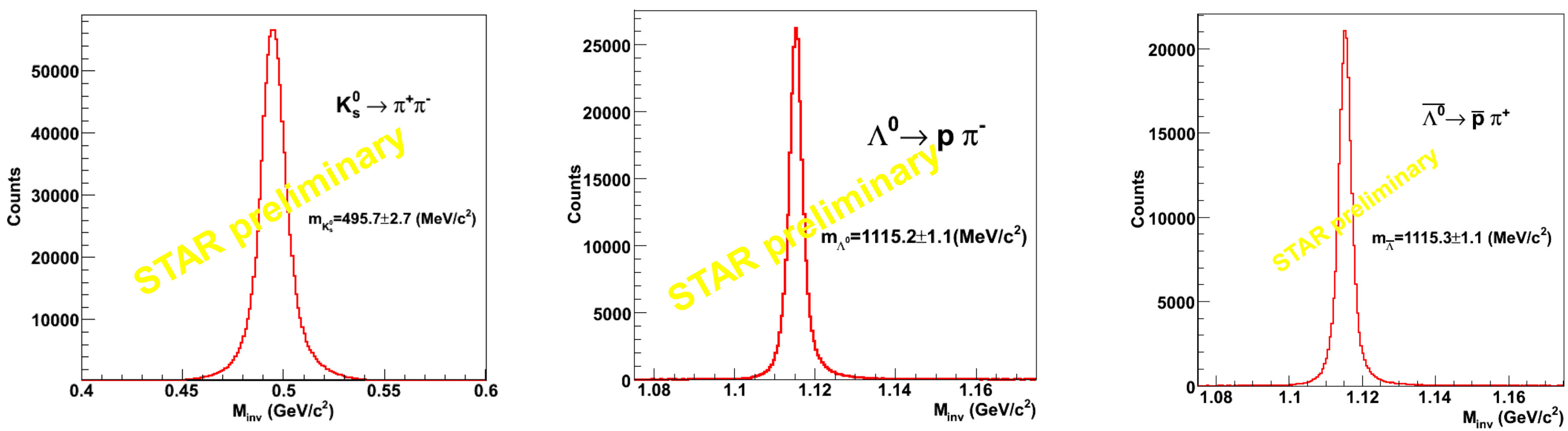
STAR Data Sample



dAu @ 200 GeV

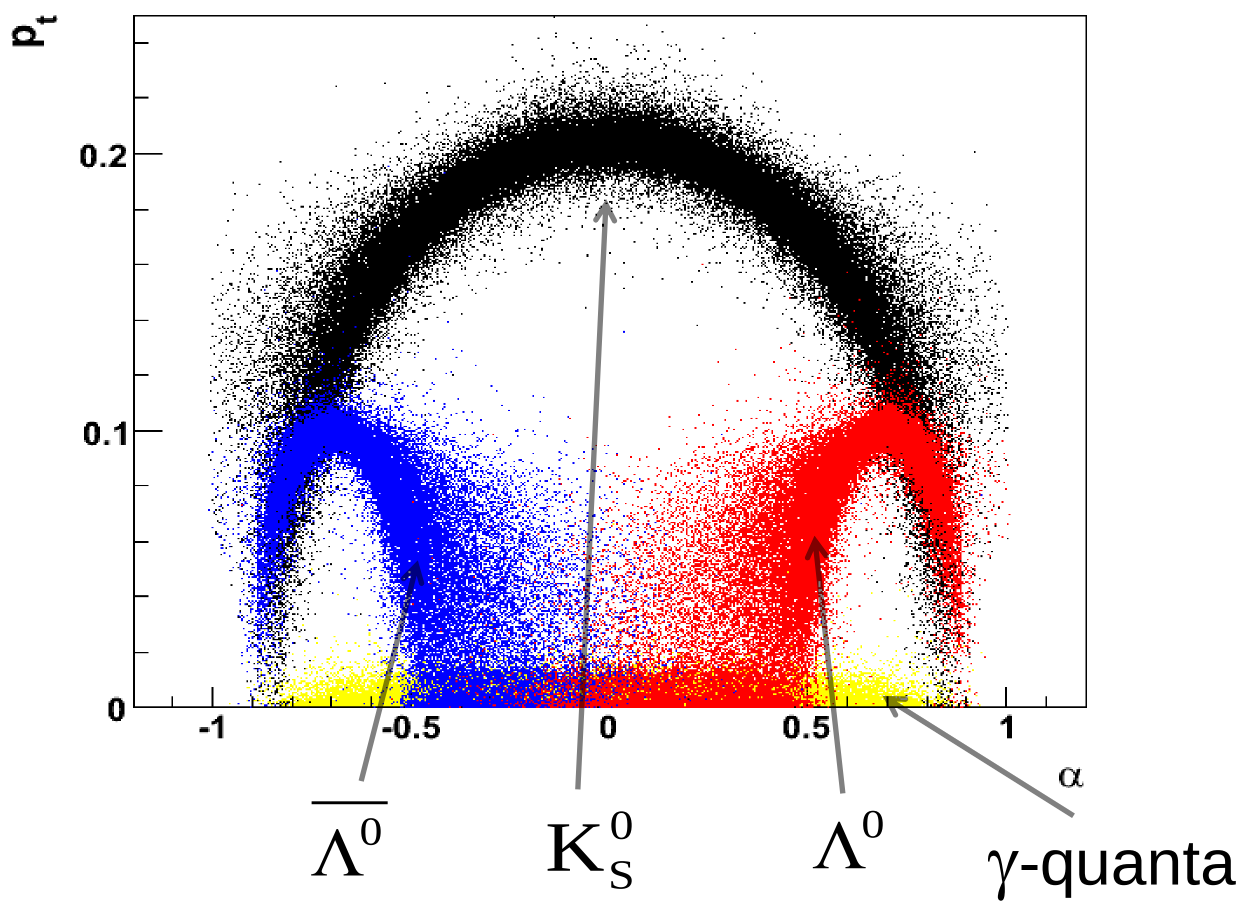
- **~13 M minbias events**
- **PID by *dE/dx*, *p/π***
- **V^0 topological cats**
- **Kinematic fitting**

particle	No. of evts
Λ^0	219K
$\overline{\Lambda}^0$	181K
K^0_S	786K



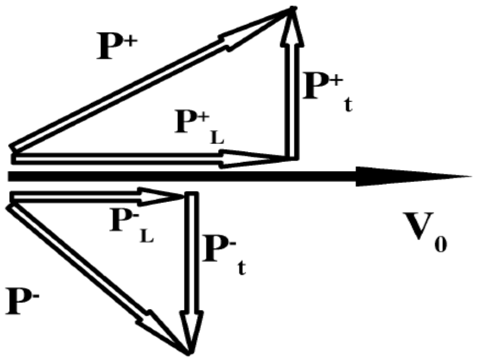
The invariant mass distributions

Armenteros-Podolanski plot



$$p_t(\alpha) = P_t^+ = P_t^-$$

$$\alpha = \frac{P_L^+ - P_L^-}{P_L^+ + P_L^-}$$

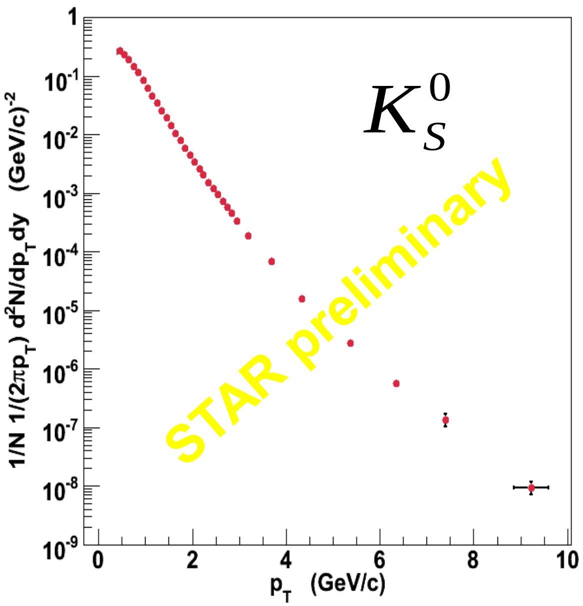


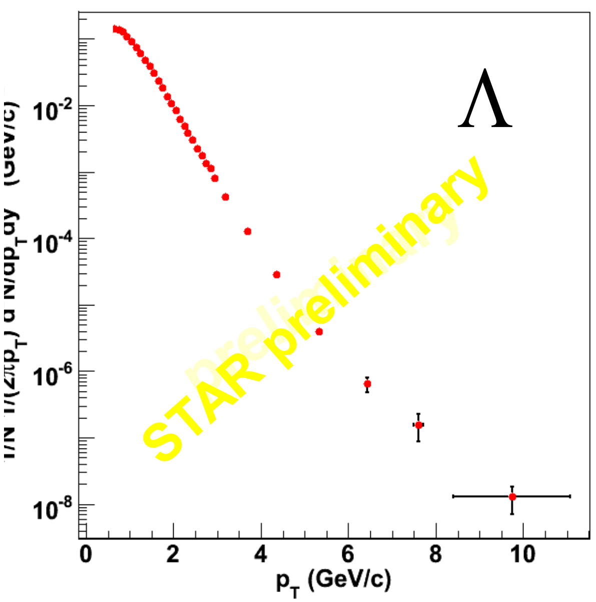
Good separation of the hypothesis

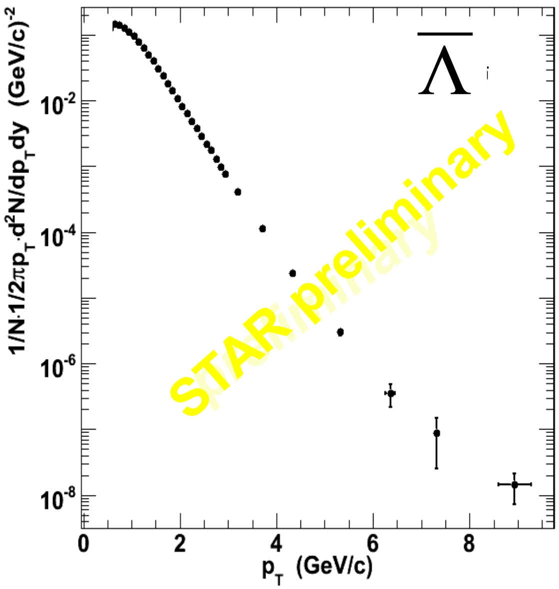
High transverse momentum spectra

dAu @ 200 GeV

$|\eta|<1$, $p_T>0.5$ GeV/c



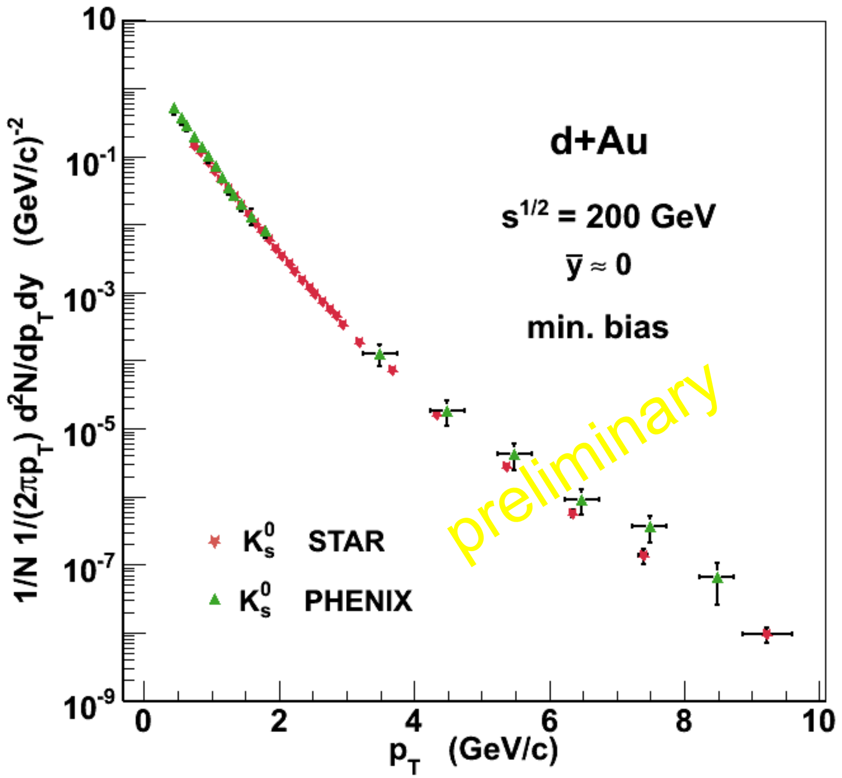


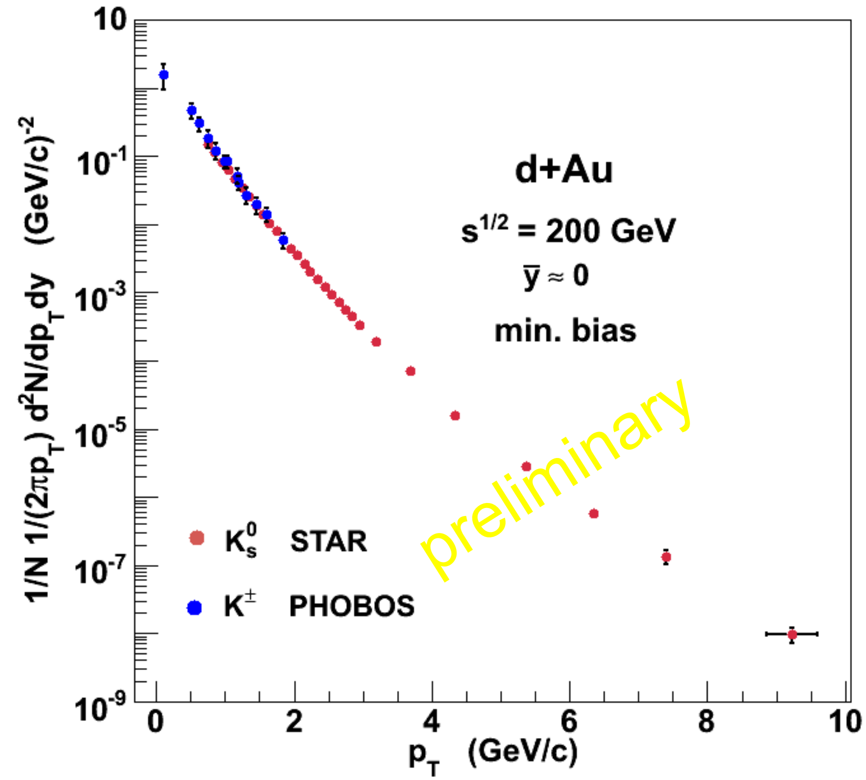


High statistics of spectra for high-p_T range

A.Kechechayn

STAR vs. PHENIX, PHOBOS spectra



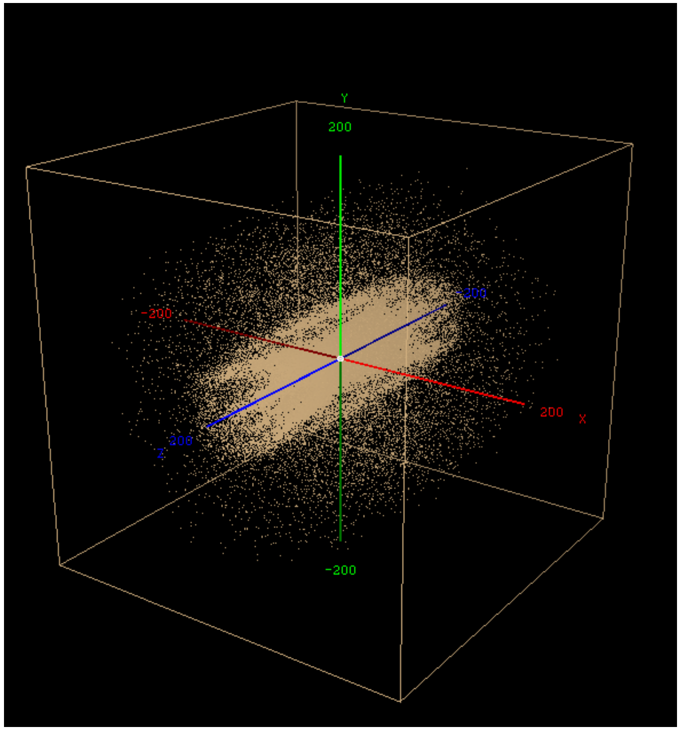


***STAR** data are in a good agreement with*
*the **PHENIX** & **PHOBOS** data in the overlapping range*

Gamma quanta & π^0

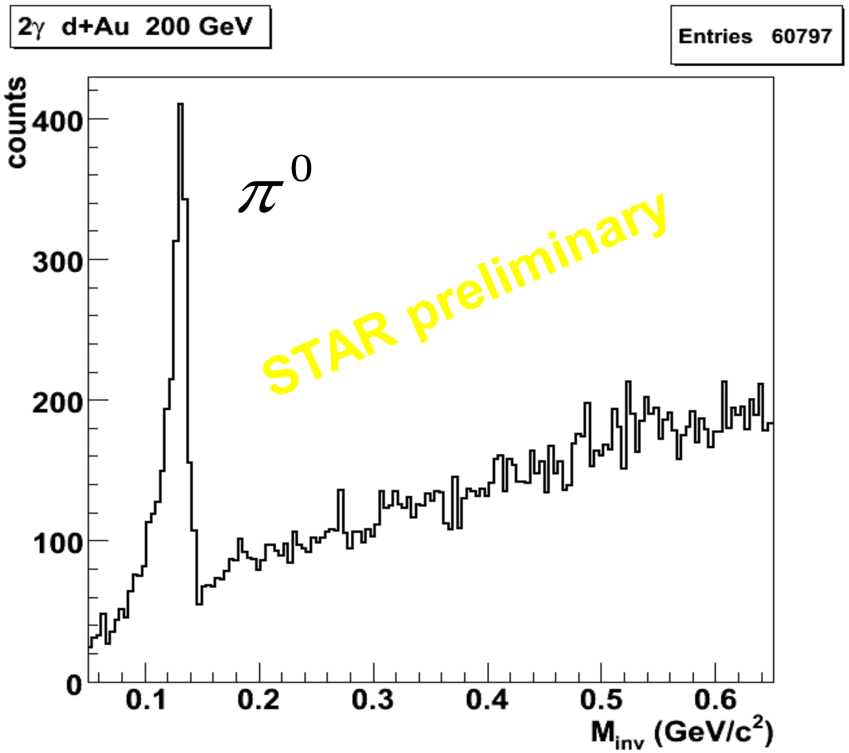
dAu @ 200 GeV

Gamma ray picture of STAR detector



x,y,z coordinates of γ conversion points

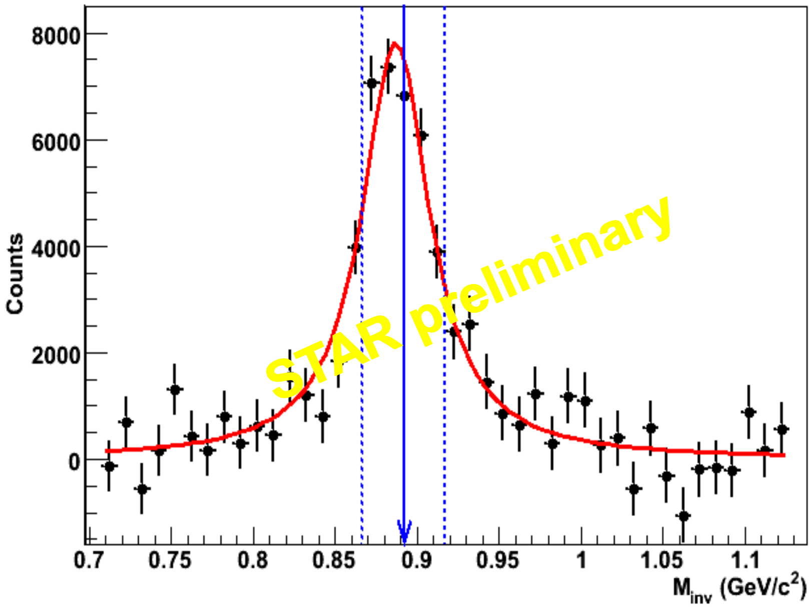
Invariant mass distribution for 2γ



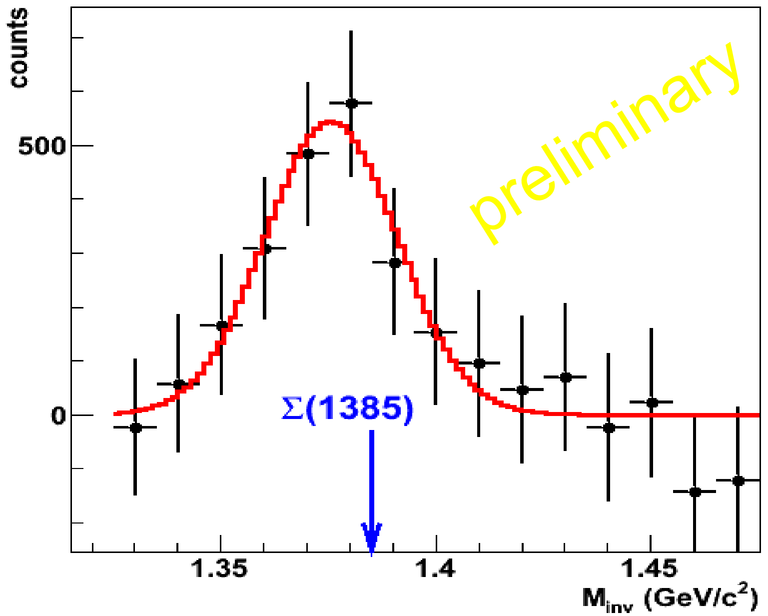
Reconstruction π^0 in TPC

Strange resonances

$K^*(892)^\pm \rightarrow K_S^0\pi^\pm$



$\Sigma(1385)^\pm \rightarrow \Lambda^0\pi^\pm$



Fit function = Non Relativistic Breit-Wigner

	Fit data	PDG data
mass	***0.8867*** ±0.0013 GeV/c^2	***0.89166*** GeV/c^2
width	***0.0498*** ±0.0051 GeV/c^2	***0.0508*** GeV/c^2

STAR

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Summary

- *Kinematic fitting was suggested and tested using MC data.*
- *This method was applied for reconstruction of* V^0 *particles from the STAR data.*
- *Transverse yields of* K_S^0, Λ^0 , $\overline{\Lambda^0}$ *particles in dAu collisions at 200 GeV at* ***high*** $\boldsymbol{p_T}$ *were obtained.*
- *Possibility of* γ *and* π^0 *reconstruction in the TPC by using the kinematic fit was shown.*
- ***The improved resolution of the V0 kinematic and geometrical parameters was obtained.***

Thanks for Your Attention !