Study of the strongly interacting matter properties with the method of factorial moments of the multiplicity distribution

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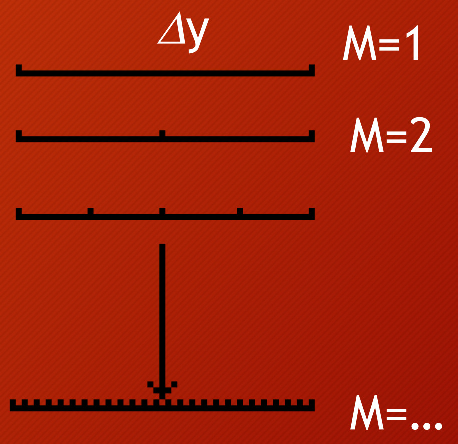
Olga Kodolova on behalf of WG?

Introduction

It was proposed by A. Bialas and R. Peschanski (Nucl. Phys. B 273 (1986) 703) to study the dependence of the normalized factorial moments

Note: there is a set of definitions of moments and cumulants.

$$F_i = M^{i-1} \times \left\langle \frac{\sum_{j=1}^{M} k_j \times (k_j - 1) \times \ldots \times (k_j - i + 1)}{N \times (N-1) \times \ldots \times (N - i + 1)} \right\rangle$$

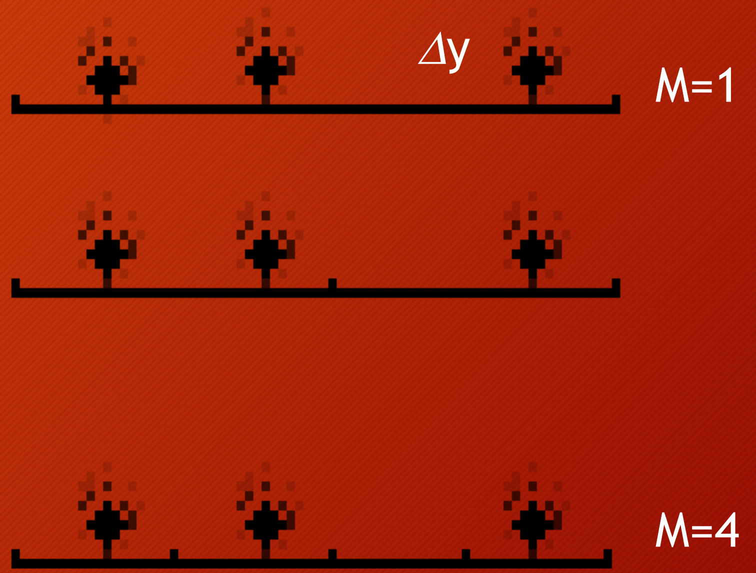


of the rapidity distribution on the size δy (Δy/M, M is the number of bins, Δy is the size of the mid rapidity window, N-number of particles in Δy, k_j-the number of particles in bin j):

1. if fluctuations are purely statistical no variation of moments as a function of δy is expected
2. Observation of variations indicates the presence of physics origin fluctuations

What do we see with factorial moments: simplified case

- Let's imagine that in each event we have an accident number of particles organized in groups and groups are distributed uniformly in Δy interval. Each group has the random number of particles. Let's imagine that all particles inside group has the same rapidity, i.e. point-like group.

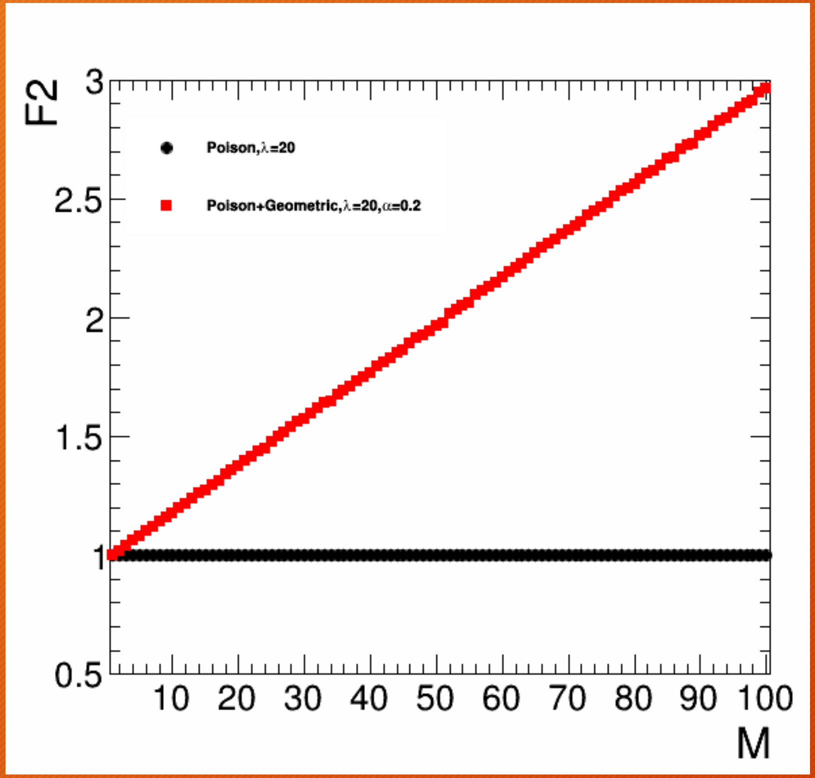


Let's the number of groups per event is Poissonian
and the number of particles per group has geometrical distribution.
Multiplicity l obeys distribution:

$$P(l) = \sum_{m=0}^{l} \frac{e^{-\lambda}\lambda^m}{m!} \binom{l-1}{m-1} \alpha^{l-m}(1-\alpha)^m$$

Under condition a<<1, this will give the same result as Negative Binomial Distribution which describes multiplicity distribution at middle interaction energy.

Simple examples



Toy events have uniform rapidity distribution in interval [-1,1]

Independent production of particles with poisonian distribution leads to Fi(M) = 1.

Under hypothesis of independent pointlike groups Fi(M) grows as polinomial of order (i-1) until the width of the rapidity distribution of the particles within group is larger than size of bin ($\sigma > \Delta y/M$).

$$F_i(M) = \sum_{k=1}^{i-1} A_k M^{k-1}$$

M.Yu.Bogolyubsky et al (11 co-authors), Clan model and factorial moments of the multiplicity distribution in intervals., Phys.Atom.Nucl. 57: 2132 - 2139, 1994

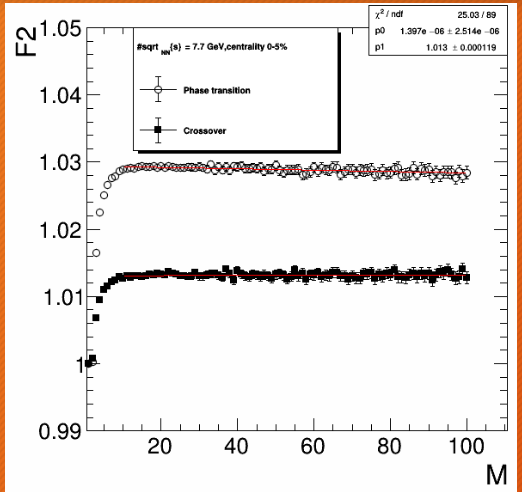
Simplified model summary

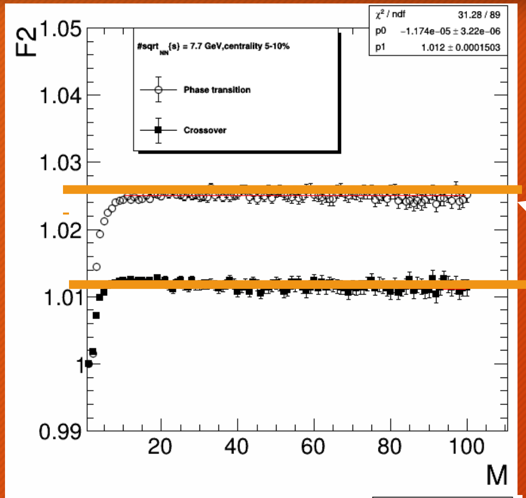
- In the case of pointlike groups (particles are produced at one rapidity point), the factorial moments grow as polinomials with increasing number *M* of bins in the partition of the initial interval.
- In the case of nonpointlike groups (particles are distributed with respect to the group center), the factorial moments at any order tends to a constant when the bin size becomes much less then characteristic width of the group.
- If several processes with different characteristic widths are effective, the factorial moments increase until the bin size becomes much less than the smallest characteristic width.
- The rate of growth depends on the density of the rapidity distribution of groups, on the multiplicity of particles in the group and on the relative probabilities of processes with different variances of particle distributions within group.

Latest studies in the world: theory and experiments

- Intermittency (fluctuations of various different sizes in 1D, 2D and 3D phase space) have been studied at LEP, Tevatron, Protvino in ee, hh, hA, AA interactions at the various energies. There are plenty of interpretations including Clan Model proposed by L. Van Hove, intermittency and fractals of the different origin.
- Some latest studies for pp and AA (NA49, NA61, ALICE):
  - A Monte Carlo Study of Multiplicity Fluctuations in Pb-Pb Collisions at LHC Energies, Ramni Gupta, Journal of Central European Green Innovation 4(4) pp 116-126 (2016)
  - Search for the critical point of strongly interacting matter in NA49 Katarzyna Grebieszkow*a* for the NA49 collaboration, arXiv:0907.4101
  - **Scaling Properties of Multiplicity Fluctuations in the AMPT Model Rohni Sharma and Ramni Gupta, AHEP, v2018, AricleID 6283801**
  - **Searching for the critical point of strongly interacting matter in nucleus-nucleus collisions at CERN SPS,** Nikolaos Davis (for the NA61/SHINE Collaboration), arXiv:2002.06636

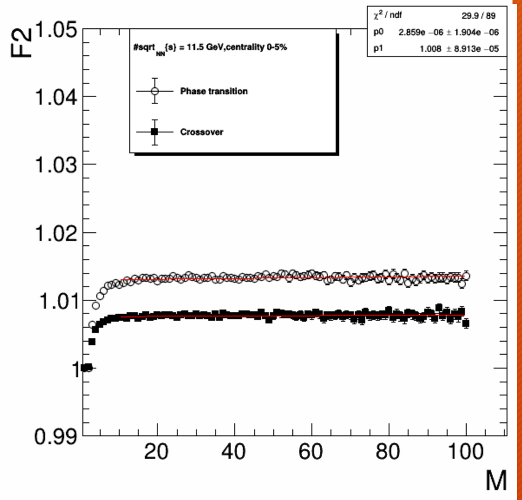
Au-Au, URQMD+VHLLE

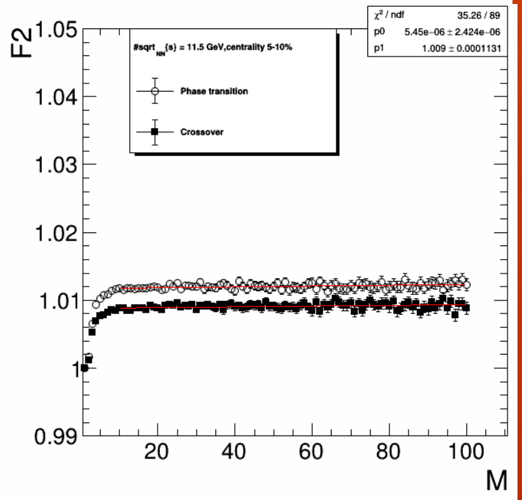




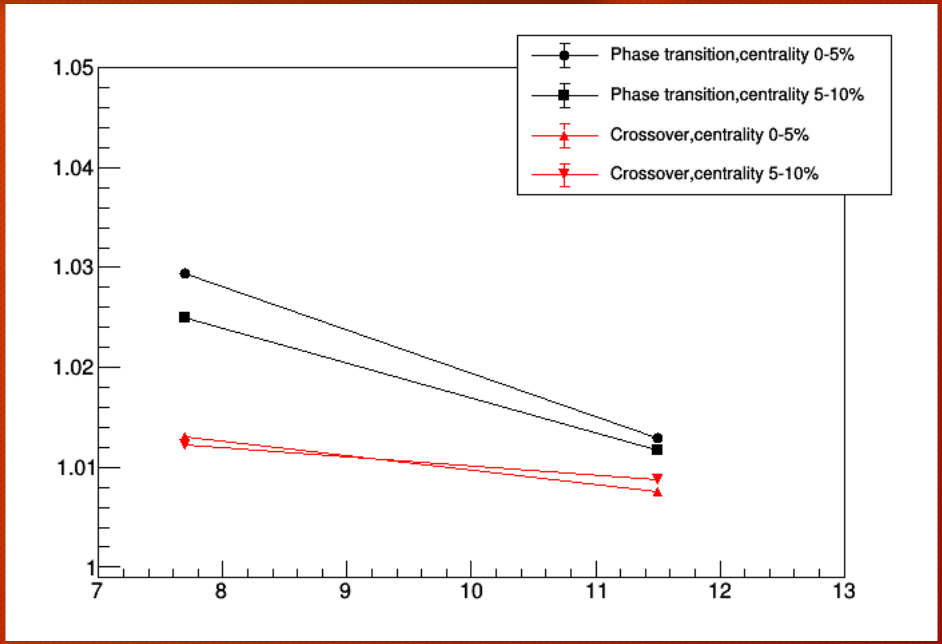
7.7 GeV

F2Max





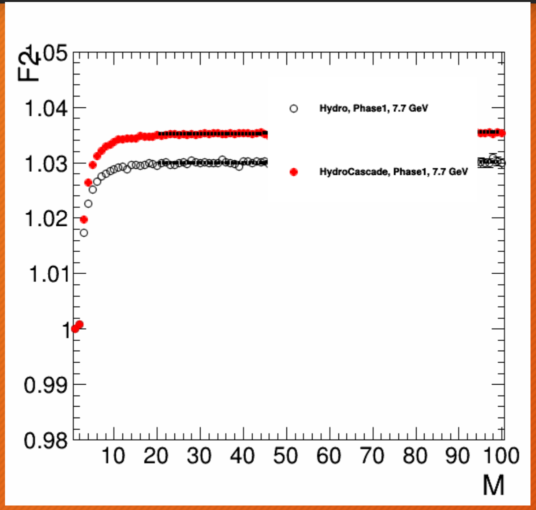
11.5 GeV

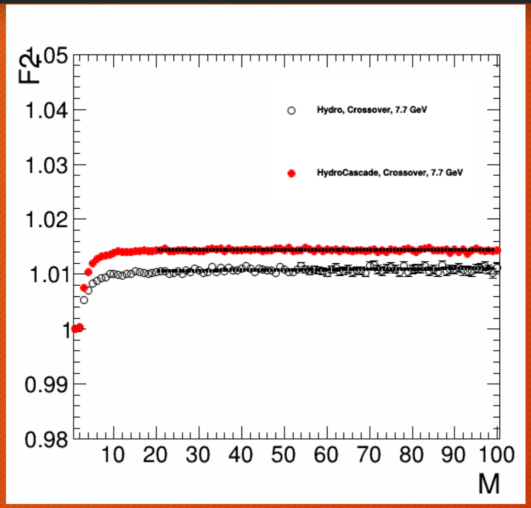


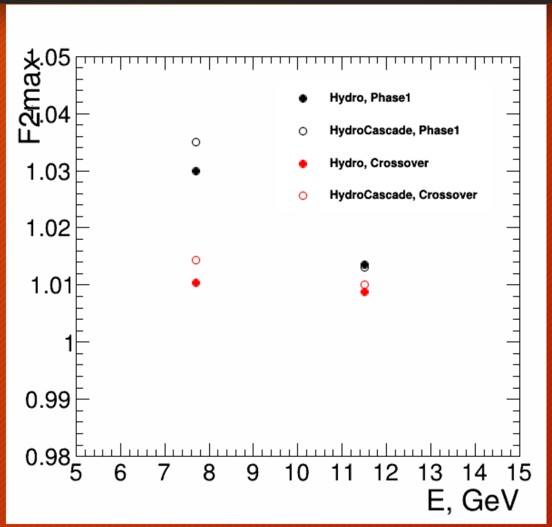
E, GeV

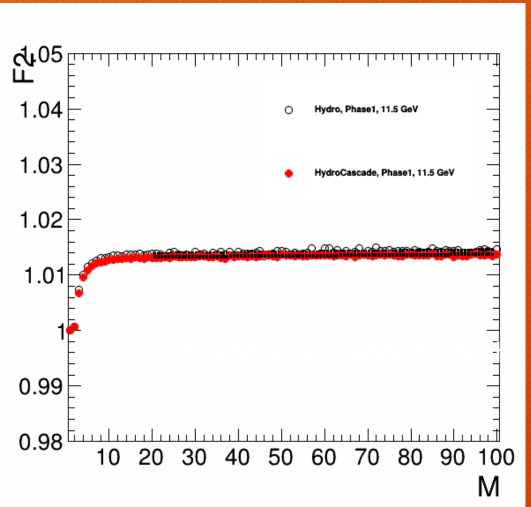
Different energy dependence is expected for Crossover and the 1st order phase transition

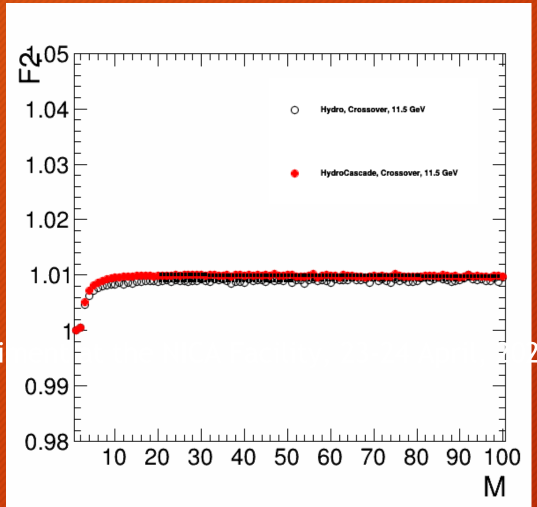
Hydro and HydroCascade separately





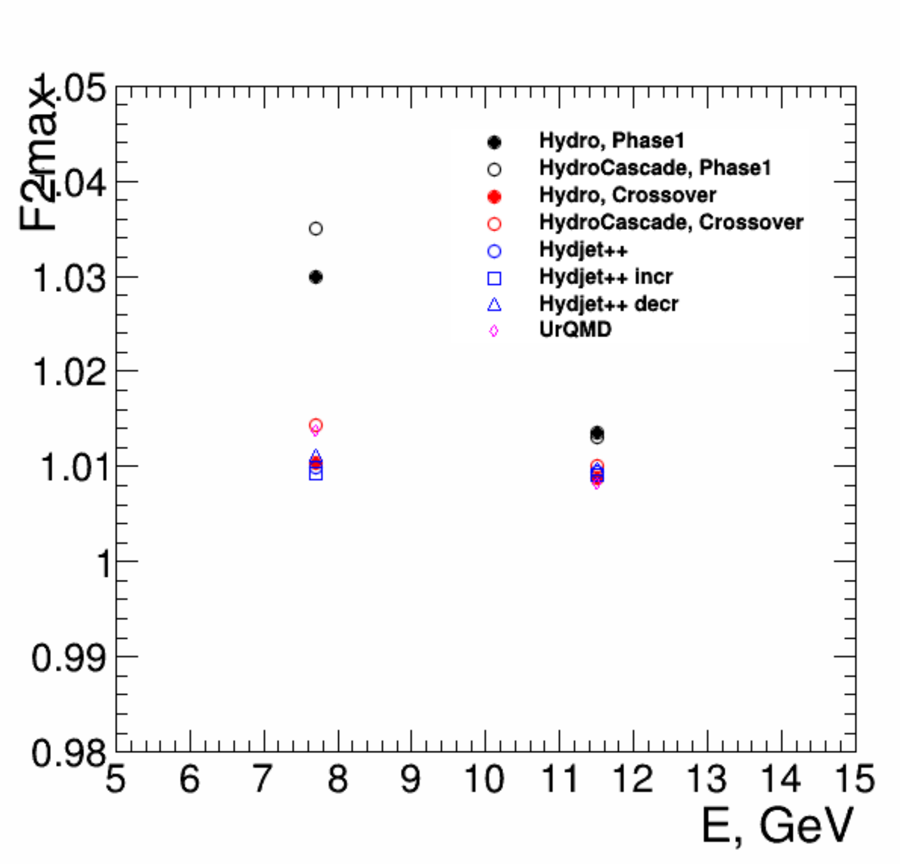






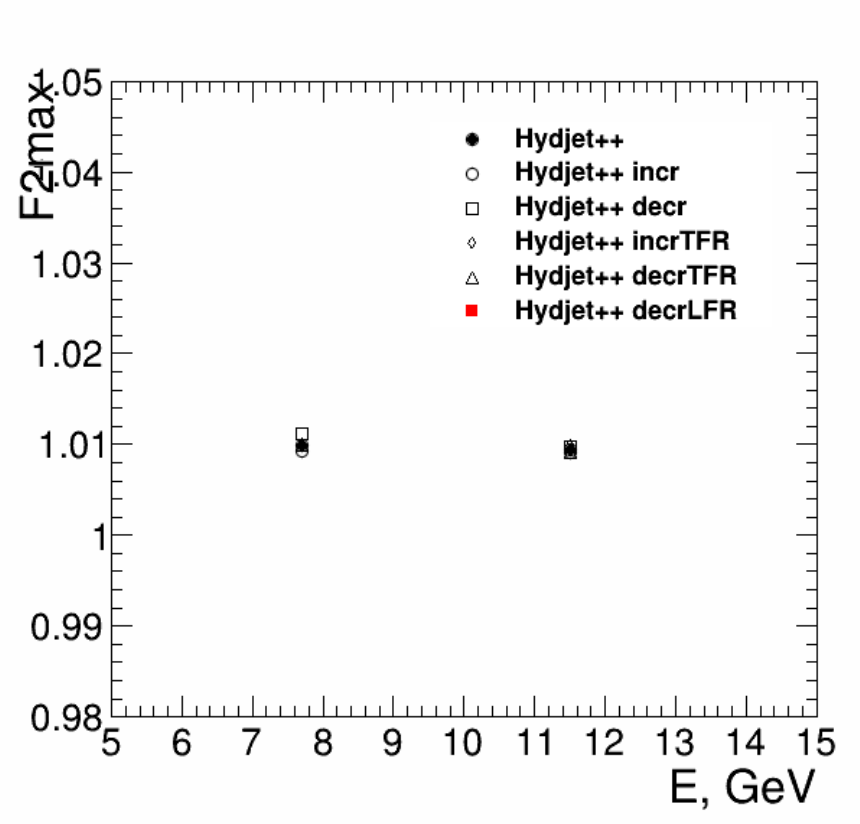
There is a small increase of the F2 maximum for HydroCascade. w.r.t Hydro only.
However the different trend in the F2 behaviour for the Phase 1 transition and crossover is visible

Models comparison:UrQMD, UrQMD+vHHLE, HYDJET++

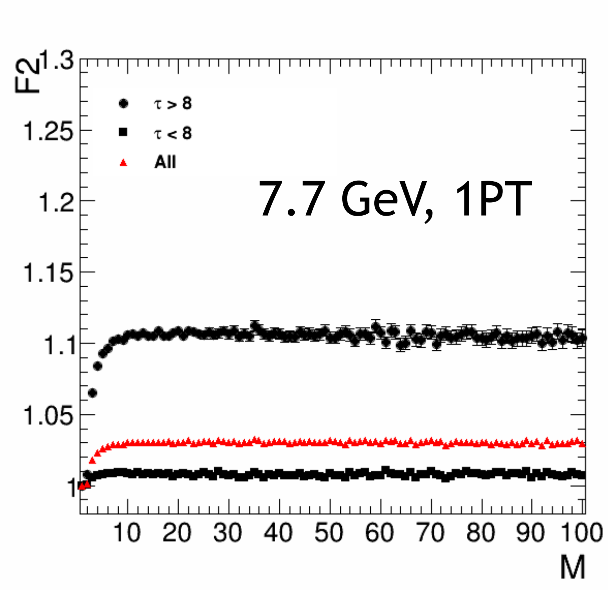


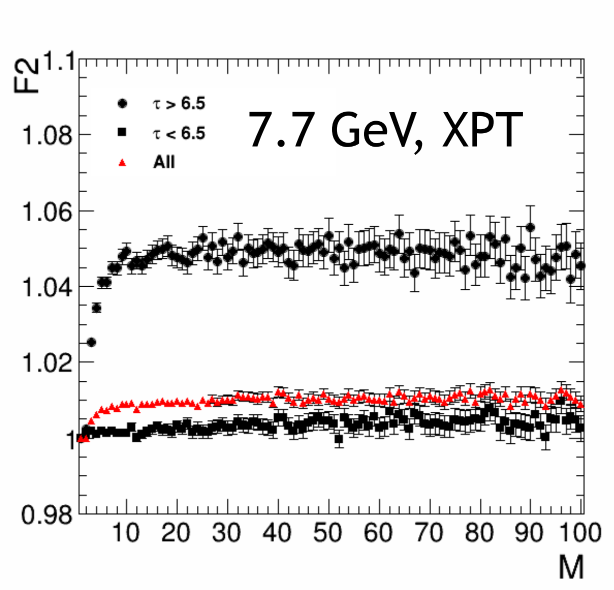
UrQMD, HYDJET++ are comparable with vHHLE+UrQMD crossover

Change of multiplicity and volume (transverse and longitudinal) size in HYDEJET++
does not affect factorial moments

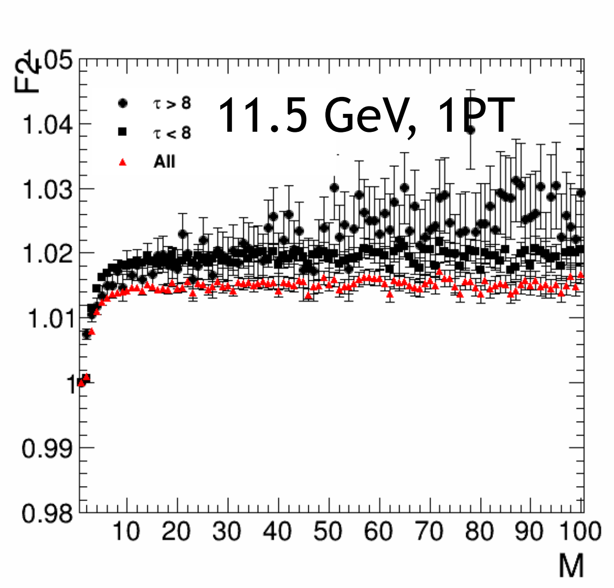


Time dependence





Interpretation is not so straightforward



Let’s consider 2 time intervals: groups are created in each of Intervals independently

$\alpha 1, \lambda 1$ $\alpha 2, \lambda 2$

$$\frac{\alpha 1}{\lambda 1} < \frac{\alpha}{\lambda 1 + \lambda 2} \qquad \frac{\alpha}{\lambda 1 + \lambda 2} < \frac{\alpha 2}{\lambda 2}$$

Conditions can NOT be fulfilled while $\alpha 1 = \alpha 2$

$\tau = \mathrm{sqrt}(t^2 + z^2)$

$\alpha 1, \lambda 1$ $\alpha 2, \lambda 2$

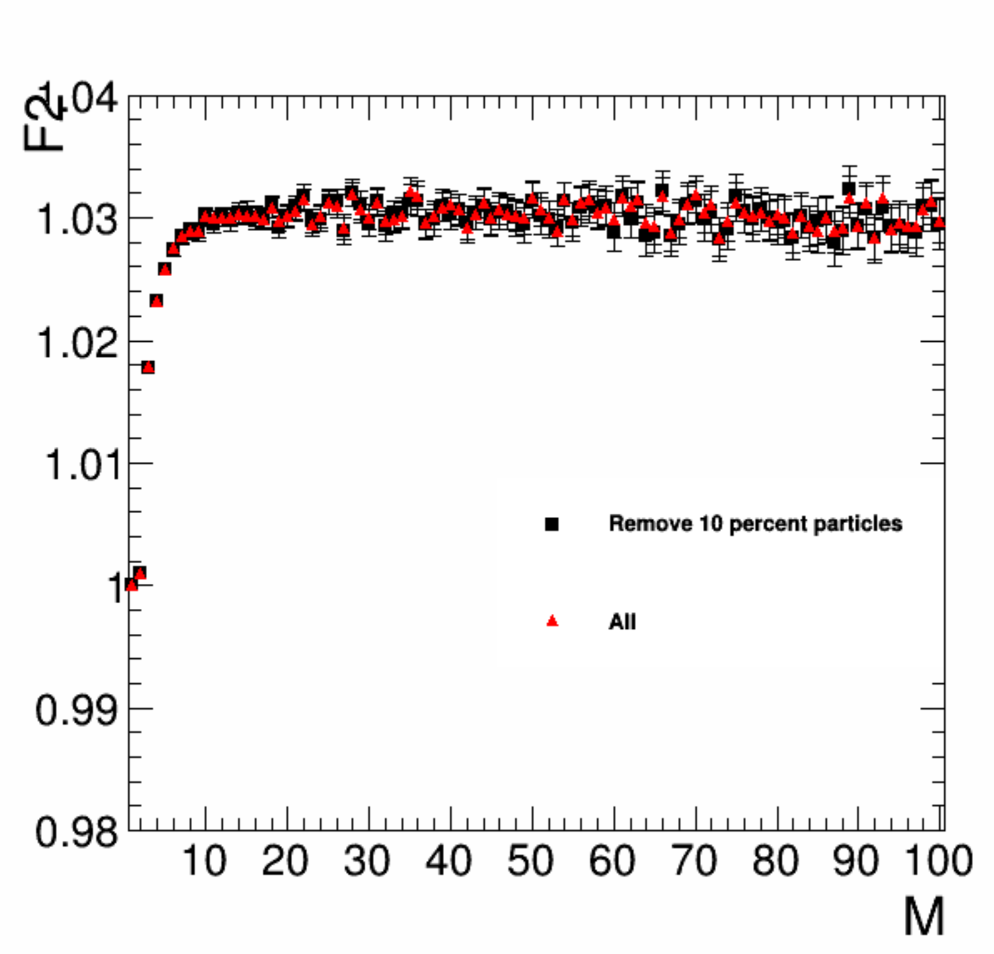
$$\frac{\alpha}{\lambda 1 + \lambda 2} < \frac{\alpha 1}{\lambda 1} \qquad \frac{\alpha}{\lambda 1 + \lambda 2} < \frac{\alpha 2}{\lambda 2}$$

Conditions can be fulfilled while $\alpha 1 = \alpha 2$

Summary

- Normalized factorial moments as a function of the size of the observation interval are sensitive to the type of phase transition.
  - We observe the different energy behaviour for the Crossover and 1^{st} order phase transition in the frame of the URQMD+VHHLE model.
  - The enregy behaviour is connected to the development of the phase transition itself. Cascade introduces the mild excess to the maximum of the normalized factorial moments.
- We start to work with reconstructed objects. The plan is to take into account the sample efficiency, purity and track momentum resolution.

Remove randomly 10% particles from events



7.7 GeV 1PT

Factorial moments addition

- Divide interval D on M bins. l-particles are distributed among m-groups
- The probability the in bin (i) we will have ν particles from m1 group under condition that l particles are in m groups

$$R = \frac{\binom{\nu-1}{m1-1}\binom{l-\nu-1}{m-m1-1}}{\binom{l-1}{m-1}}$$

- Under fixed l,m,m1

$$G_i(l,m,m1) = M\sum_{\nu=m1}^{l-(m-m1)} \frac{\nu*(\nu-1)*\cdots*(\nu-i+1)}{l*(l-1)*\cdots*(l-i+1)} \frac{\binom{\nu-1}{m1-1}\binom{l-\nu-1}{m-m1-1}}{\binom{l-1}{m-1}}$$

Factorial moments addition

Let’s assume the unform distribution of groups along interval y

$$F_i = M^{i-1} \sum_{m=0}^{\infty} \frac{e^{-\lambda}\lambda^m}{m!} \sum_{l=m}^{\infty} \binom{l-1}{m-1} \alpha^{l-m} (1-\alpha)^m \sum_{m1=0}^{m} \binom{m}{m1} \left(\frac{1}{M}\right)^{m1} \left(1-\frac{1}{M}\right)^{m-m1} G_i\ (\mathrm{l,m,m1})$$

After calculations:

$$F_i(M) = \sum_{k=1}^{i-1} A_k M^{k-1}$$

For F2:

$$A_1 = 1 - \frac{2\alpha(1-e^{-\lambda})}{\lambda} \qquad A_2 = \frac{2\alpha(1-e^{-\lambda})}{\lambda}$$