Factorial moments in NICA MPD experiment

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Introduction

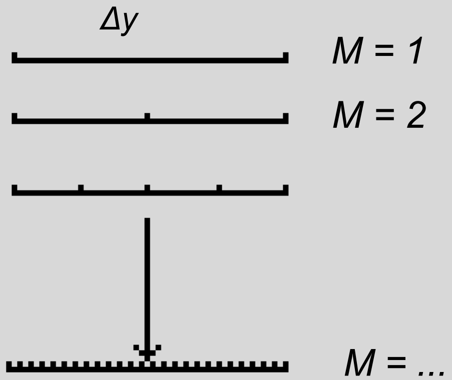
It was proposed by A. Bialas and R. Peschanski (Nucl. Phys. B 273 (1986) 703) to study the dependence of the normalized factorial moments of the rapidity distribution on the bin size δ*y*:

1. if fluctuations are purely statistical no variation of moments as a function of δ*y* is expected

2. observation of variations indicates the presence of physics origin fluctuations

$$F_i = M^{i-1} \times \left\langle \frac{\sum_{j=1}^{M} k_j \times (k_j - 1) \times \ldots \times (k_j - i + 1)}{N \times (N-1) \times \ldots \times (N - i + 1)} \right\rangle$$

$\delta y = \Delta y / M$
M — number of bins
Δy — size of mid rapidity window
N — number of particles in Δy
k_j-the number of particles in bin j



<u>Note:</u> there is a set of definitions of moments and cumulants.

What do we see with factorial moments: simplified case

Mathematical model:

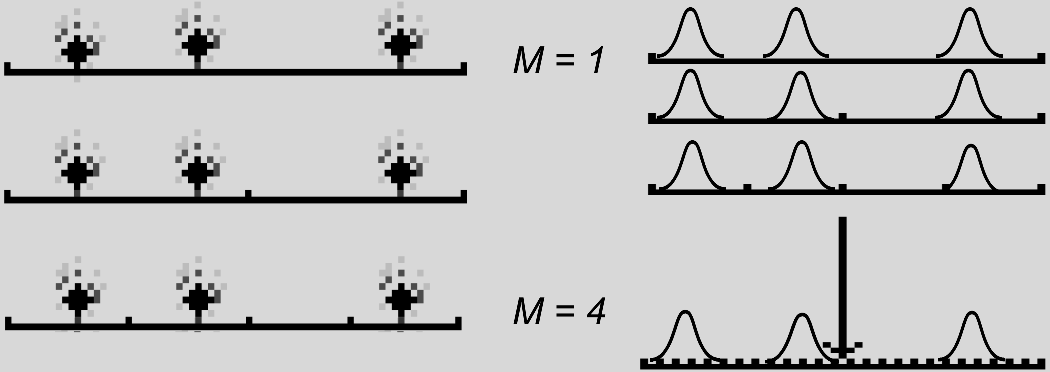
- **an accident number of particles per event organized in groups**
- **groups are distributed uniformly along *Δy* interval.**
- **Each group has the random number of particles.**
- **Consider two cases:**
  - **Point-like group** - all particles inside group has the same y,
  - **Non point-like group** - particles are distributed over y with respect to the group center

- number of groups per event is Poissonian
- number of particles per group has geometrical distribution.

Multiplicity distributions of particles in Δy interval is:

$$P(l) = \sum_{m=0}^{l} \frac{e^{-\lambda}\lambda^m}{m!} \binom{l-1}{m-1} \alpha^{l-m}(1-\alpha)^m$$

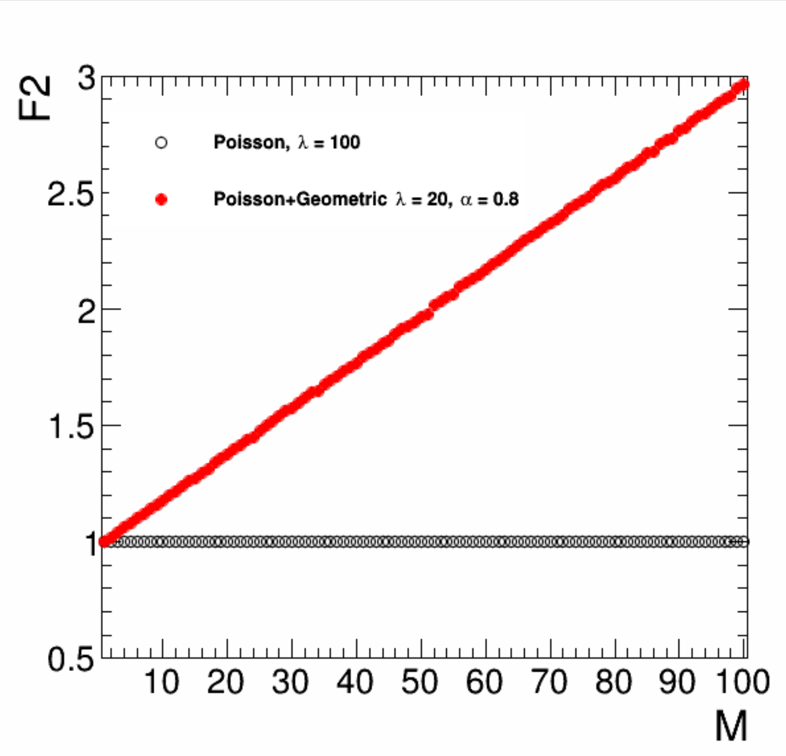
Under condition ***α<<1***, this will give **the same result as Negative Binomial Distribution** which describes multiplicity distribution at middle interaction energy.



Point-like groups

Nonpoint-like groups with width σ

Simple examples: pointlike groups



Toy events have uniform rapidity distribution in interval [-1,1]

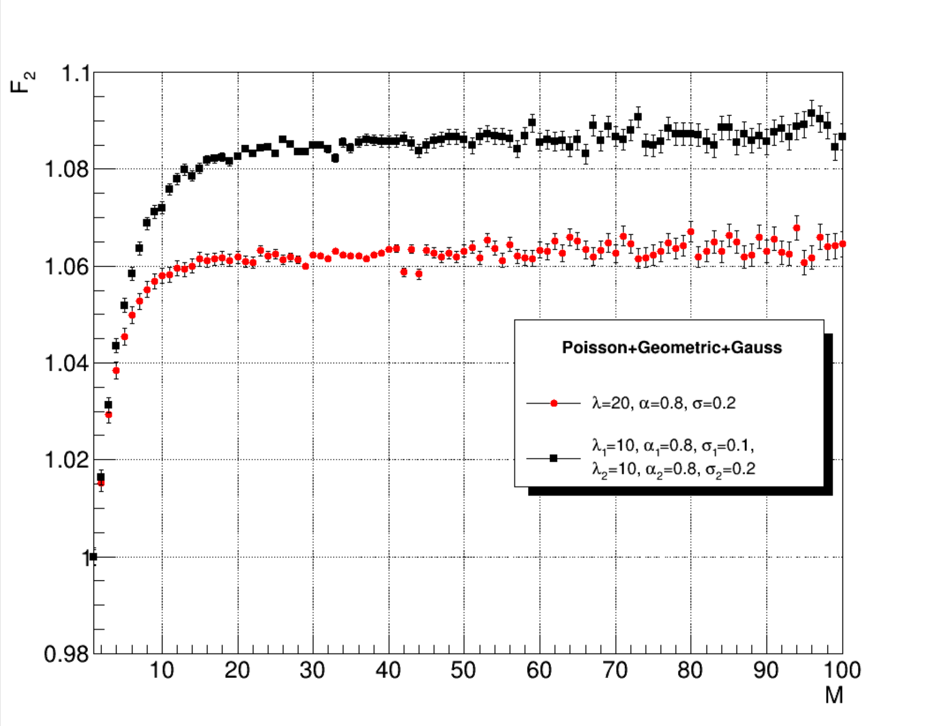
Independent production of particles with poisonian distribution leads to $F_i(M) = 1$.

Under hypothesis of independent pointlike groups $F_i(M)$ grows as polinomial of order (i-1)

$$F_i(M) = \sum_{k=1}^{i-1} A_k M^{k-1}$$

M.Yu.Bogolyubsky et al (11 co-authors), Clan model and factorial moments of the multiplicity distribution in intervals., Phys.Atom.Nucl. 57: 2132 - 2139, 1994

Simple examples: non-pointlike groups



- **For non-pointlike group** with width σ.

F_i (M) = constant when $\delta y = \Delta Y/M \ll \sigma$

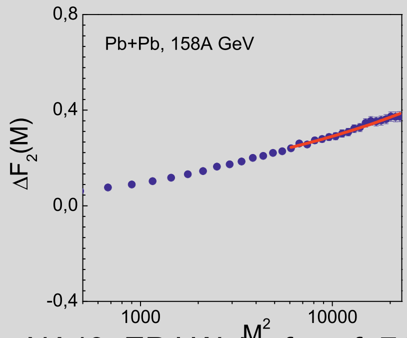
- **Several processes with different characteristic widths** ($\sigma_1 > \sigma_2 > .. > \sigma_N$) the factorial moments are increasing untill $\delta y = \Delta Y/M \ll \sigma_N$

- The power of growth depends on
  - Mean number of groups
  - Mean number of particles per group
  - Characteristic widths of groups

Latest studies of intermittency in the world: theory and experiments

- Intermittency have been studied at LEP, Tevatron, Protvino in ee, hh, hA, AA interactions at the various energies. There are plenty of interpretations including Clan Model proposed by L. Van Hove, intermittency and fractals of the different origin.
- Recent studies in NA49, NA61

$$\Delta F_2 = F_2^{data} - F_2^{mixed} \propto M^{2\varphi}$$



NA49, EPJ Web of conf, 71,

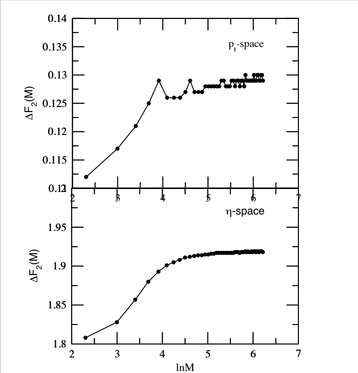
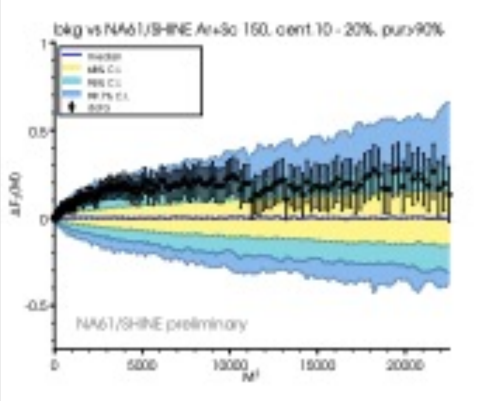


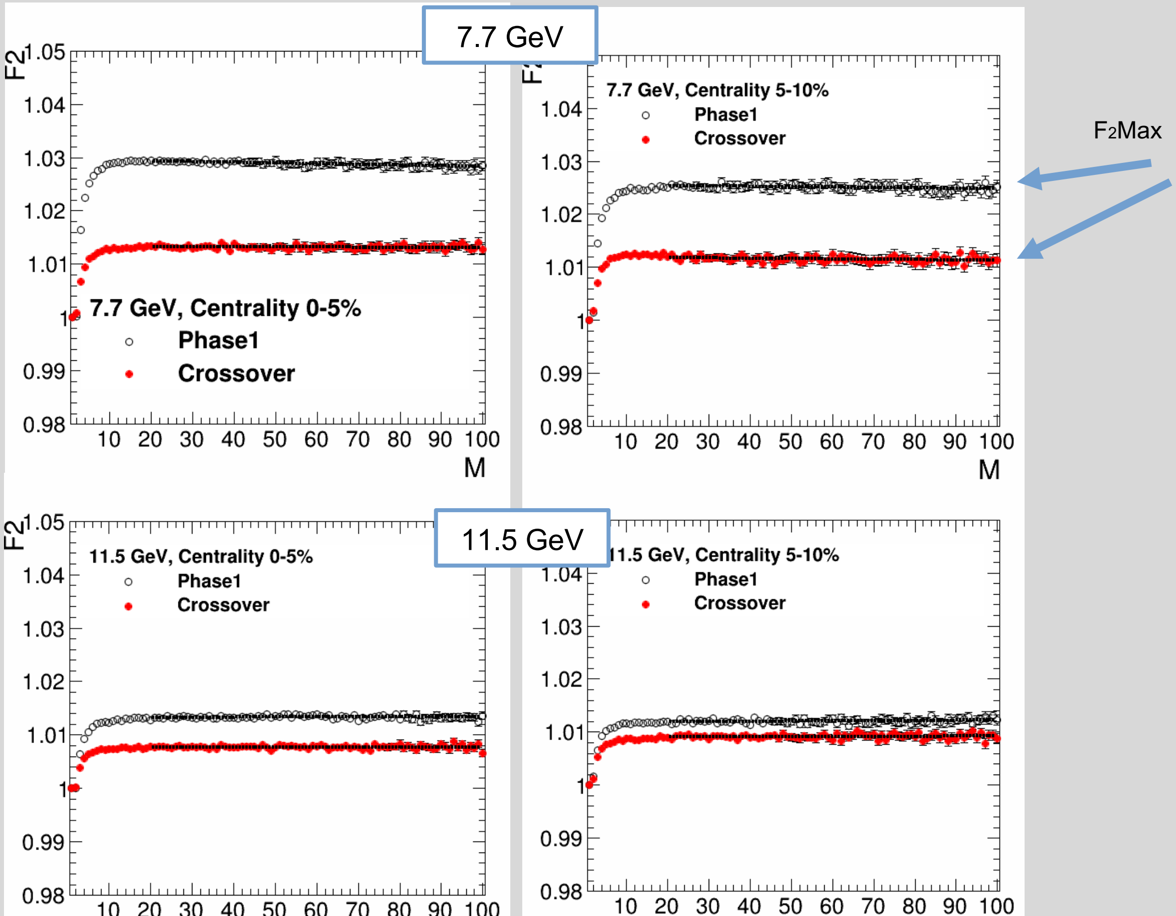
Fig.1 Variation of $\Delta F_2(M)$ with natural logarith of number of bins, M for AMPT simulated Pb+Pb collisions at 2.76 TeV per nucleon pair energy.

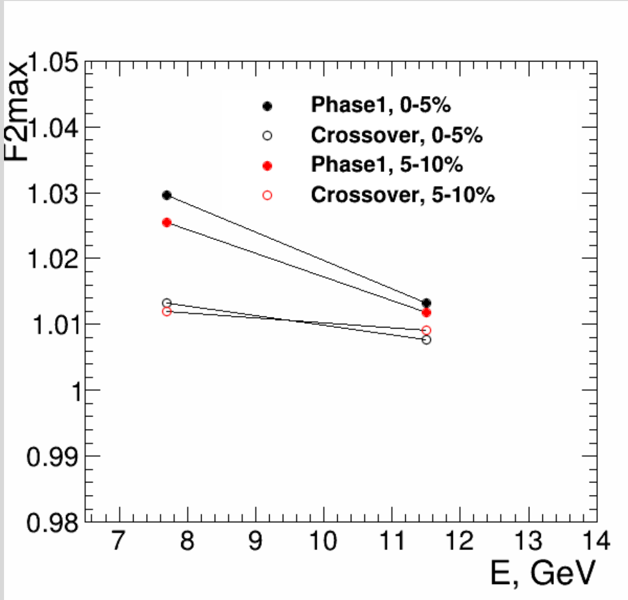
M.M.Khan et al, DAE-BRNS Symp. On Nucl.Phys. 61(2016)



NA61/SHINE, arXiv:2002.06636,2020

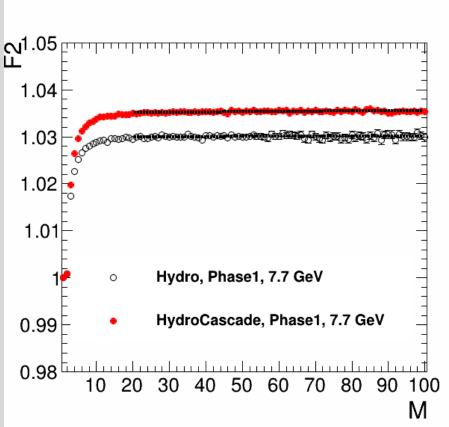
Au-Au, UrQMD+vHLLE

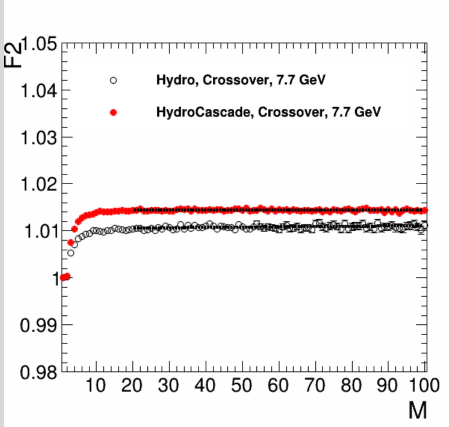


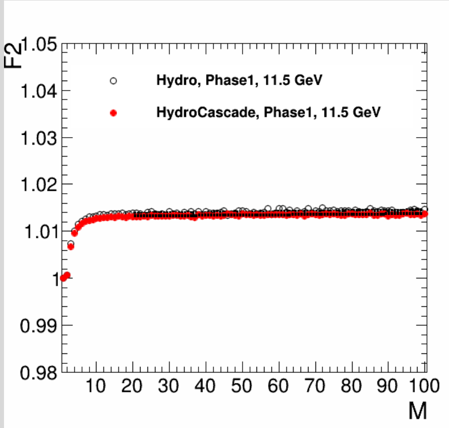


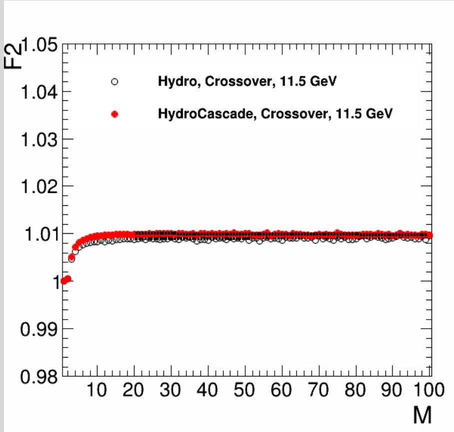
- Different energy dependence is expected for Crossover and 1st order phase transition
- There is a mild dependence on centrality for 1st order

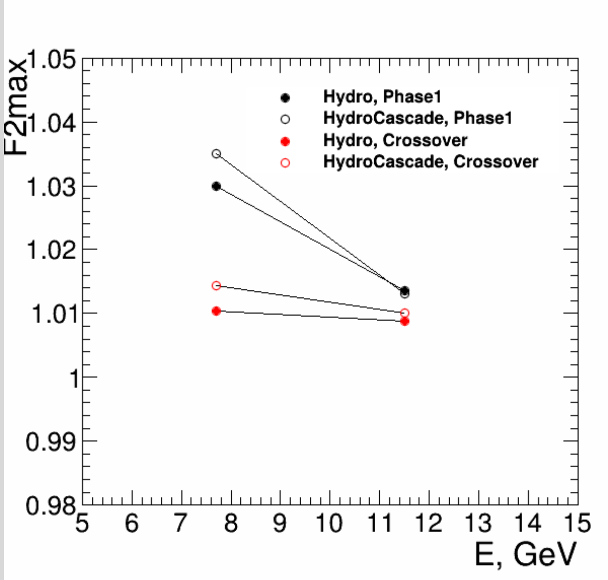
Hydro and HydroCascade separately





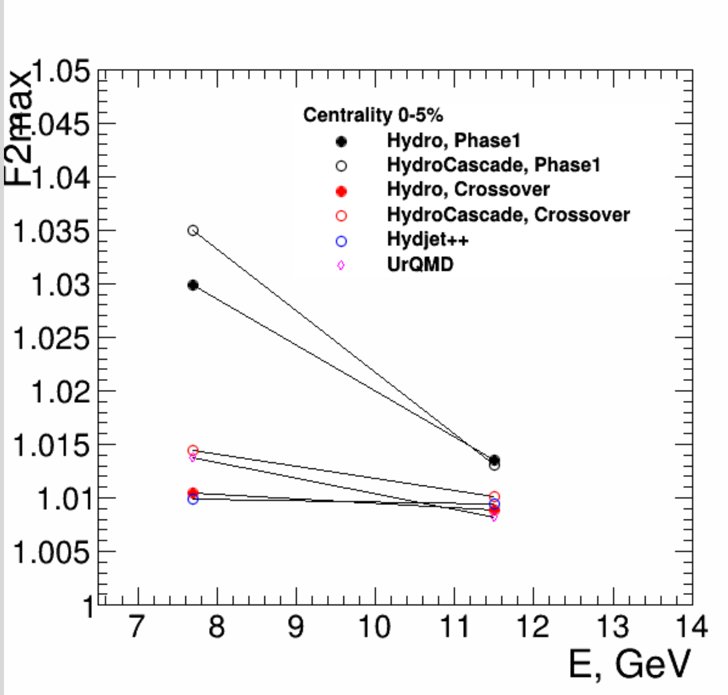




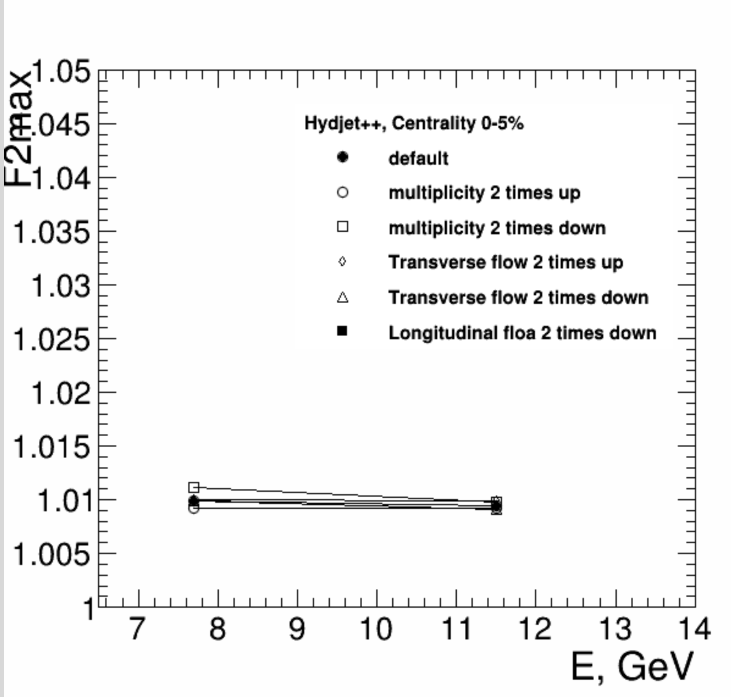


- There is a small increase of the F_2 maximum for HydroCascade. w.r.t Hydro only.
- However the different trend in the F_2 behaviour for the Phase 1 transition and crossover is visible

Models comparison: UrQMD, UrQMD+vHLLE, HYDJET++



- UrQMD, HYDJET++ are comparable with vHLLE+UrQMD crossover
- Change of
  - Multiplicity
  - volume size



Summary

- Normalized factorial moments as a function of the size of the observation interval are sensitive to the type of phase transition.
  - We observe the different energy behaviour for the Crossover and 1st order phase transition in the frame of the URQMD+VHLLE model.
  - The energy behaviour is connected to the development of the phase transition and hydrodynamical phase itself. Cascade introduces the mild excess to the maximum of the normalized factorial moments.
- We start to work with reconstructed objects. The plan is to take into account the sample efficiency, purity and track momentum resolution.

Factorial moments addition

- Divide interval *D* on *M* bins. *l* – particles are distributed among *m*-groups.
- The probability in the bin (*i*) we will have *n* particles from *m1* group under condition that *l* particles are in *m* groups:

$$R = \frac{\binom{\nu-1}{m1-1}\binom{l-\nu-1}{m-m1-1}}{\binom{l-1}{m-1}}$$

- Under fixed *l*, *m*, *m1*:

$$G_i(l,m,m1) = M \sum_{\nu=m1}^{l-(m-m1)} \frac{\nu * (\nu-1) * \cdots * (\nu-i+1)}{l * (l-1) * \cdots * (l-i+1)} \frac{\binom{\nu-1}{m1-1}\binom{l-\nu-1}{m-m1-1}}{\binom{l-1}{m-1}}$$

Factorial moments addition

- Let's assume the uniform distribution of groups along interval *y*

$$F_i = M^{i-1} \sum_{m=0}^{\infty} \frac{e^{-\lambda}\lambda^m}{m!} \sum_{l=m}^{\infty} \binom{l-1}{m-1} \alpha^{l-m} (1-\alpha)^m \sum_{m1=0}^{m} \binom{m}{m1} \left(\frac{1}{M}\right)^{m1} \left(1-\frac{1}{M}\right)^{m-m1} G_i\,(\mathrm{l,m,m1})$$

- After calculation:

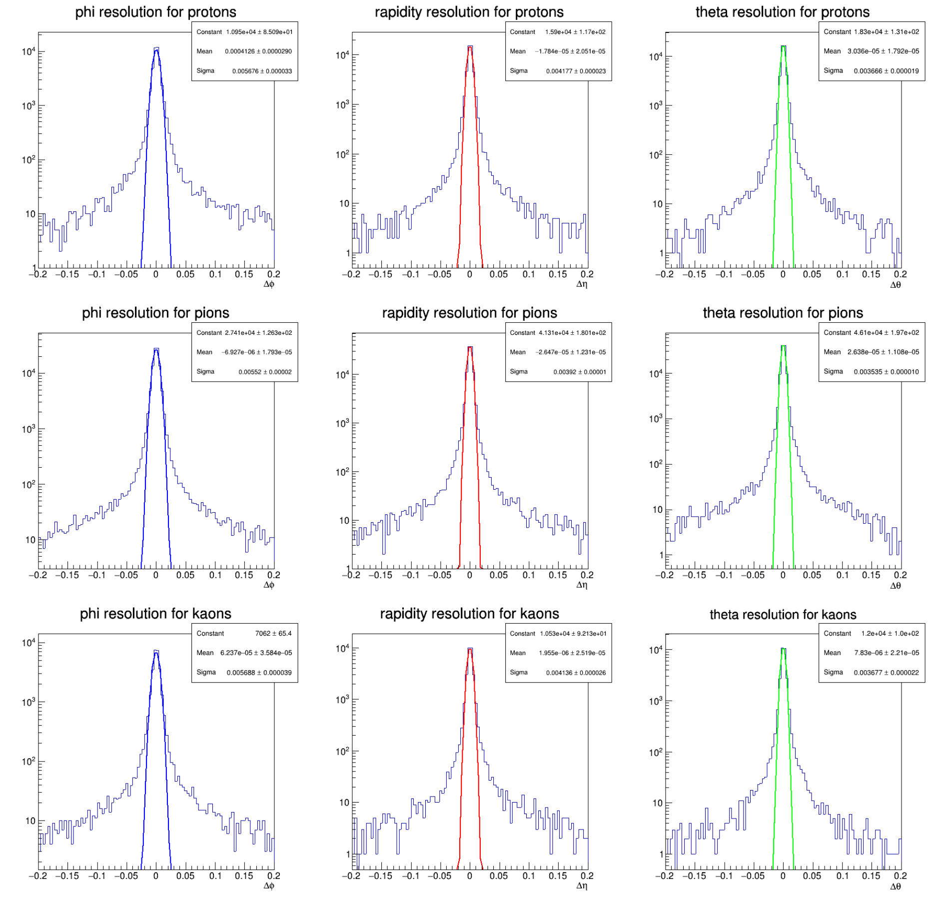
$$F_i(M) = \sum_{k=1}^{i-1} A_k M^{k-1}$$

- For F_2:

$$A_1 = 1 - \frac{2\alpha(1-e^{-\lambda})}{\lambda}$$

$$A_2 = \frac{2\alpha(1-e^{-\lambda})}{\lambda}$$

Resolution of detector



Latest studies in the world: theory and experiments

- Intermittency (fluctuations of various different sizes in 1D, 2D and 3D phase space) have been studied at LEP, Tevatron, Protvino in ee, hh, hA, AA interactions at the various energies. There are plenty of interpretations including Clan Model proposed by L. Van Hove, intermittency and fractals of the different origin

Some latest studies for pp and AA (NA49, NA61, ALICE):

- A Monte Carlo Study of Multiplicity Fluctuations in Pb-Pb Collisions at LHC Energies, Ramni Gupta, Journal of Central European Green Innovation 4(4) pp 116-126 (2016)
- Search for the critical point of strongly interacting matter in NA49 Katarzyna Grebieszkow*a* for the NA49 collaboration, arXiv:0907.4101
- Scaling Properties of Multiplicity Fluctuations in the AMPT Model Rohni Sharma and Ramni Gupta, AHEP, v2018, AricleID 6283801
- Searching for the critical point of strongly interacting matter in nucleus-nucleus collisions at CERN SPS, Nikolaos Davis