Study of strongly interacting matter properties at the energies of the NICA collider using the methods of factorial moments

Olga Kodolova, Mariya Cheremnova, Pavel Batyuk,
Ludmila Malinina, Konstantin Mikhaylov, Grigory Nigmatkulov

Introduction

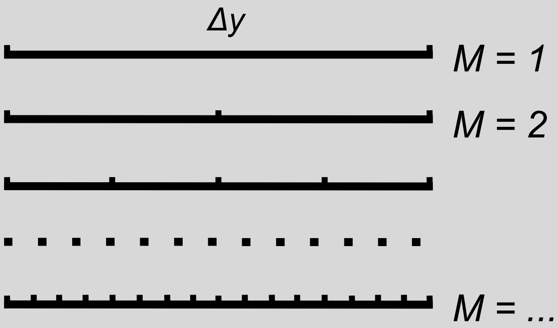
It was proposed by A. Bialas and R. Peschanski (Nucl. Phys. B 273 (1986) 703) to study the dependence of the normalized factorial moments of the rapidity distribution on the size δ*y*:

1. if fluctuations are purely statistical no variation of moments as a function of δ*y* is expected

2. observation of variations indicates the presence of physics origin fluctuations

$$F_i = M^{i-1} \times \left\langle \frac{\sum_{j=1}^{M} k_j \times (k_j - 1) \times \ldots \times (k_j - i + 1)}{N \times (N-1) \times \ldots \times (N - i + 1)} \right\rangle$$

$\delta y = \Delta y / M$
M — number of bins
Δy — size of mid rapidity window
N — number of particles in Δy
k_j-the number of particles in bin j



Note: there is a set of definitions of moments and cumulants.

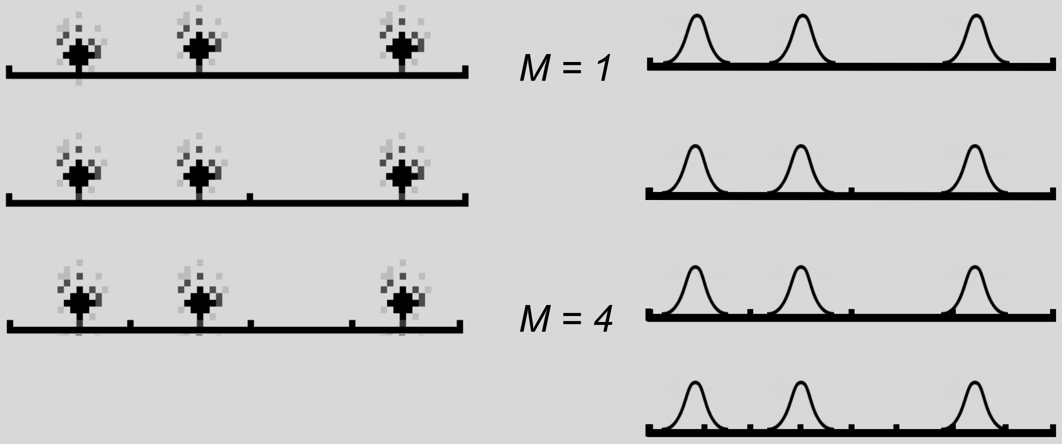
What do we see with factorial moments: simplified case

Let's imagine that in each event we have an accident number of particles organized in groups and groups are distributed uniformly in Δy interval. Each group has the random number of particles. Let's imagine that all particles inside group has the same rapidity, i.e. point-like group in first case. In the second case particles are distributed with respect to the group center – nonpointlike groups.

Let's the number of groups per event is Poissonian and number of particles per group has geometrical distribution.
Multiplicity obeys distribution:

$$P(l) = \sum_{m=0}^{l} \frac{e^{-\lambda}\lambda^m}{m!} \binom{l-1}{m-1} \alpha^{l-m}(1-\alpha)^m$$

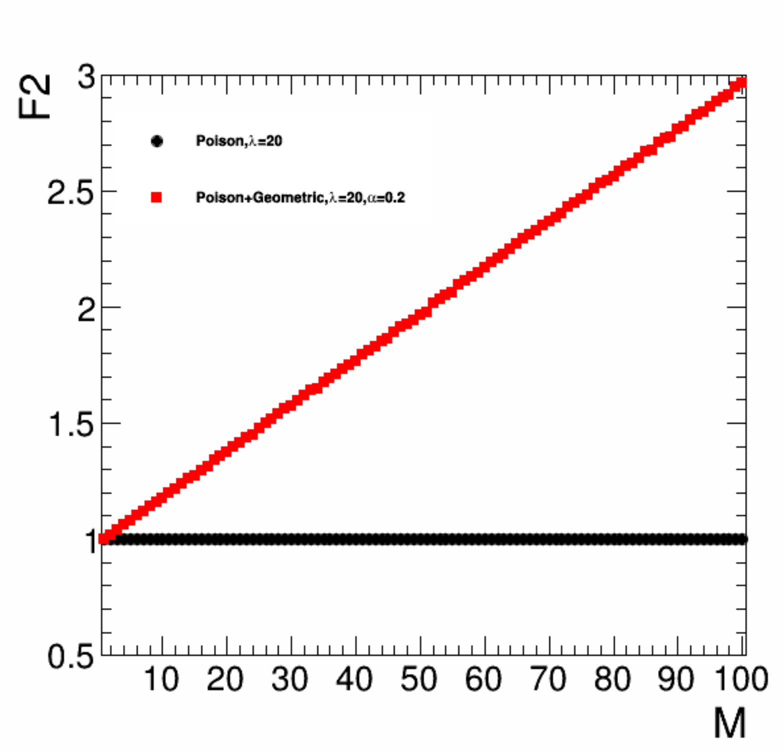
Under condition *a<<1*, this will give the same result as Negative Binomial Distribution which describes multiplicity distribution at middle interaction energy.



Point-like groups

Nonpoint-like groups

Simple examples



Toy events have uniform rapidity distribution in interval [-1,1]

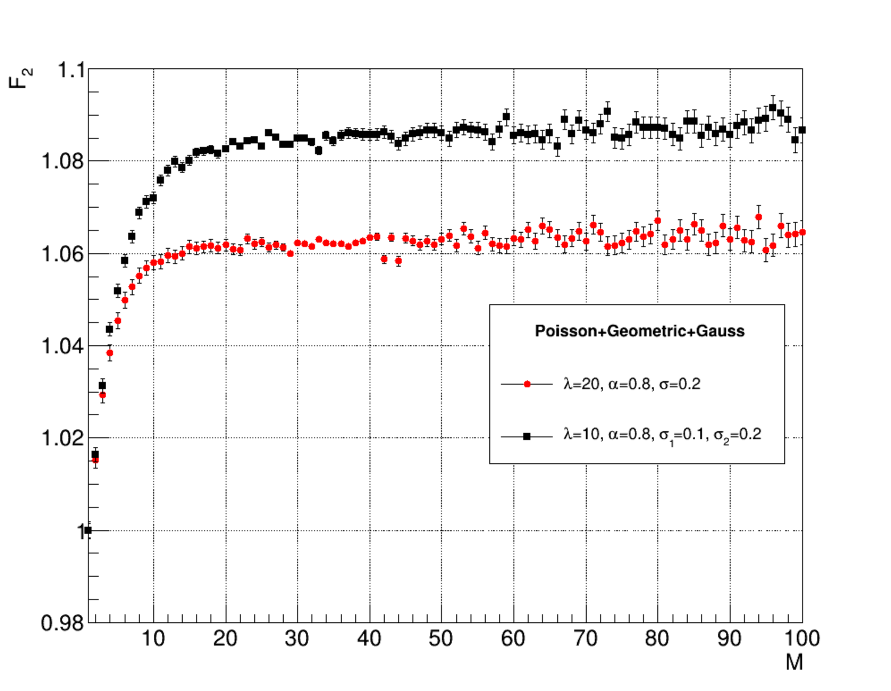
Independent production of particles with poisonian distribution leads to $F_i(M) = 1$.

Under hypothesis of independent pointlike groups $F_i(M)$ grows as polinomial of order ($i-1$) until the width of the rapidity distribution of the particles within group is larger than size of bin ($\sigma > \Delta y/M$).

$$F_i(M) = \sum_{k=1}^{i-1} A_k M^{k-1}$$

M.Yu.Bogolyubsky et al (11 co-authors), Clan model and factorial moments of the multiplicity distribution in intervals., Phys.Atom.Nucl. 57: 2132 - 2139, 1994

Simple examples

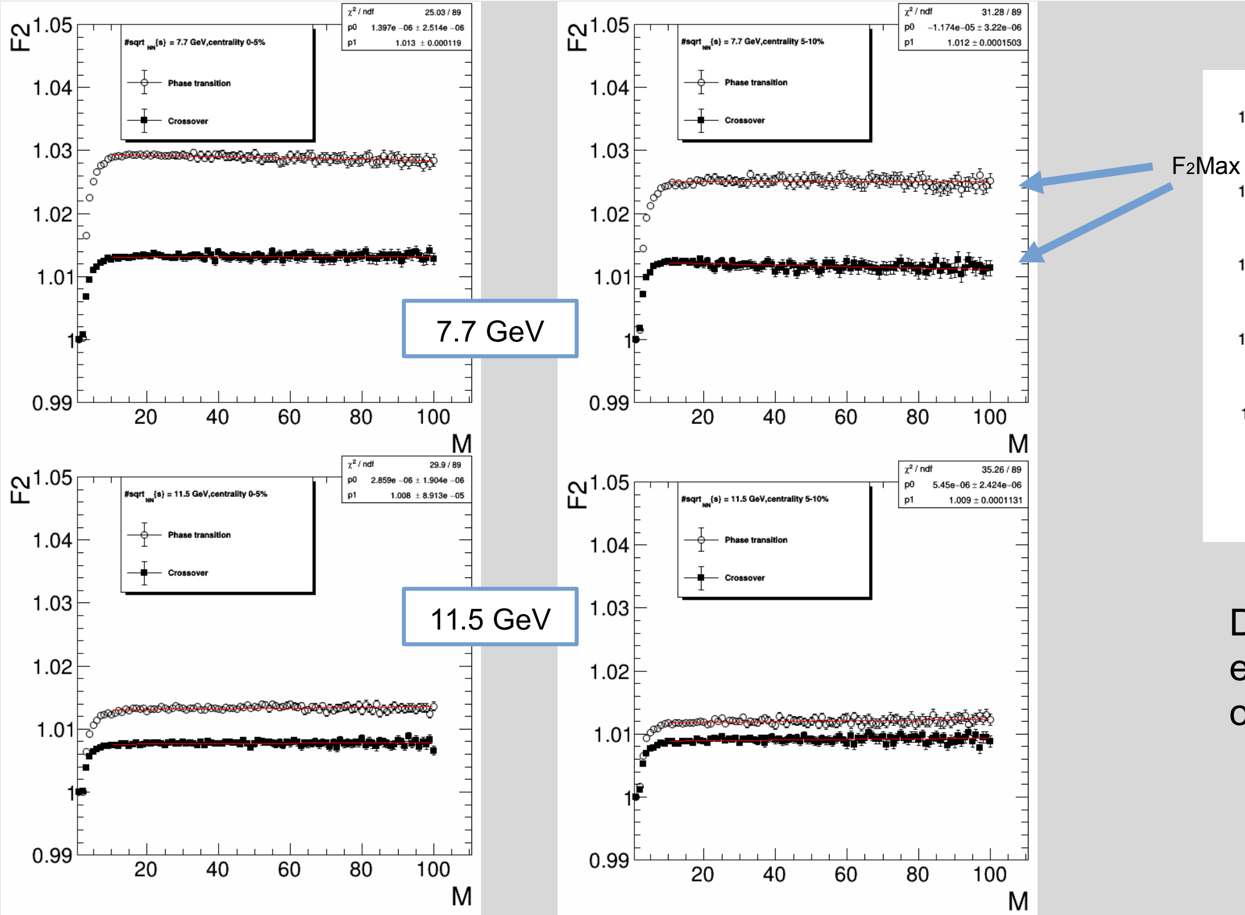


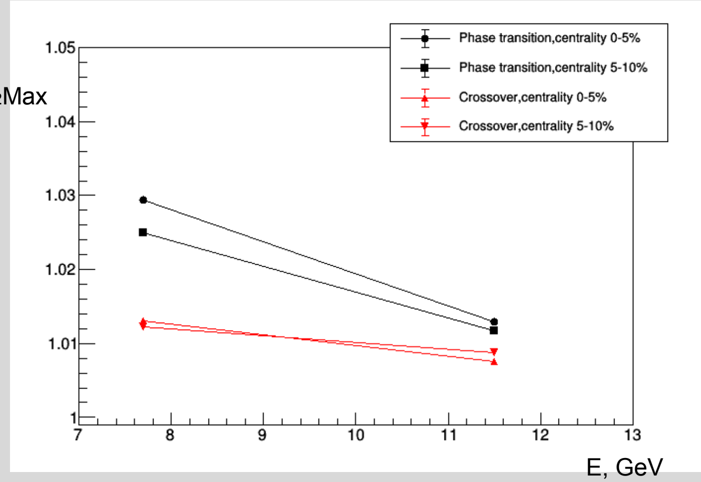
- In the case of nonpointlike groups the factorial moments at any order tends to a constant when the bin size becomes much less then characteristic width of the group.
- If several processes with different characteristic widths are effective, the factorial moments are increase until the bin size becomes much less than the smallest characteristic width.
- The rate of growth depends of destiny of the rapidity distribution of groups, on the multiplicity of particles in the group and on the relative probabilities of processes with different variances of particle distributions within group.

Latest studies in the world: theory and experiments

- Intermittency (fluctuations of various different sizes in 1D, 2D and 3D phase space) have been studied at LEP, Tevatron, Protvino in ee, hh, hA, AA interactions at the various energies. There are plenty of interpretations including Clan Model proposed by L. Van Hove, intermittency and fractals of the different origin.
- Some latest studies for pp and AA (NA49, NA61, ALICE):
  - A Monte Carlo Study of Multiplicity Fluctuations in Pb-Pb Collisions at LHC Energies, Ramni Gupta, Journal of Central European Green Innovation 4(4) pp 116-126 (2016)
  - Search for the critical point of strongly interacting matter in NA49 Katarzyna Grebieszkowa for the NA49 collaboration, arXiv:0907.4101
  - Scaling Properties of Multiplicity Fluctuations in the AMPT Model Rohni Sharma and Ramni Gupta, AHEP, v2018, AricleID 6283801
  - Searching for the critical point of strongly interacting matter in nucleus-nucleus collisions at CERN SPS, Nikolaos Davis

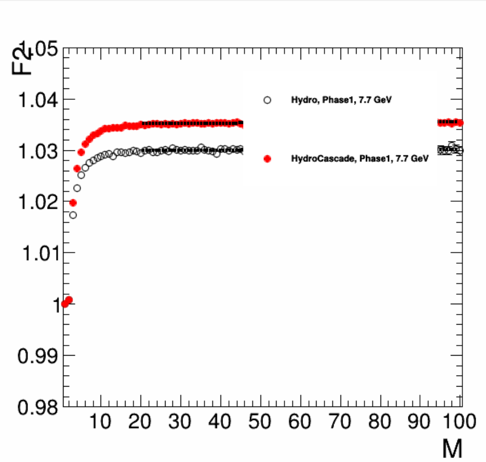
Au-Au, UrQMD+vHLLE

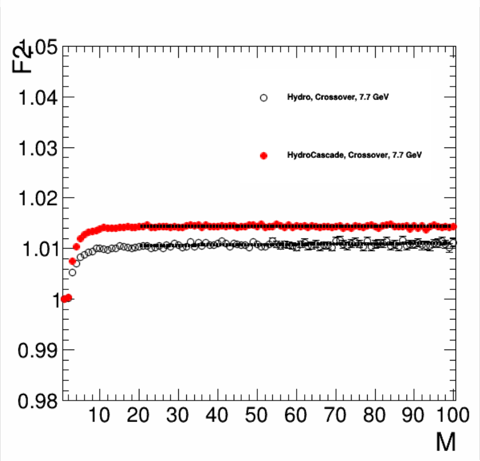


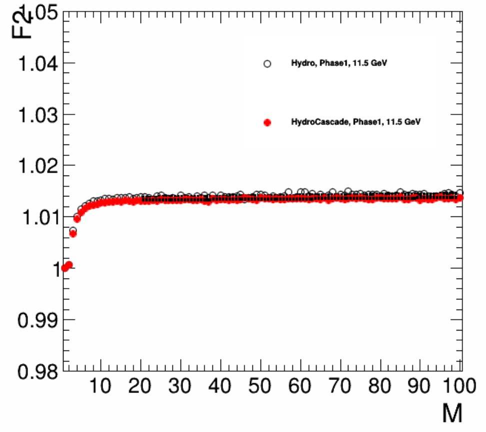


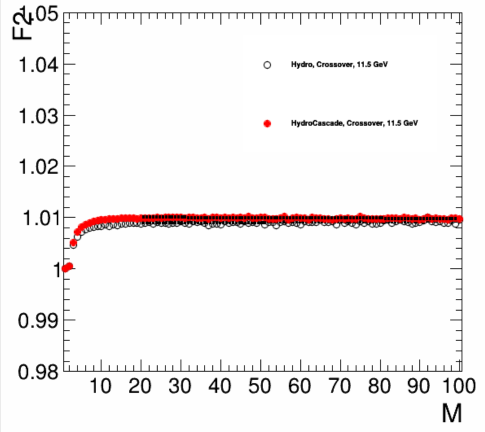
Different energy dependence is expected for Crossover and 1st order phase transition

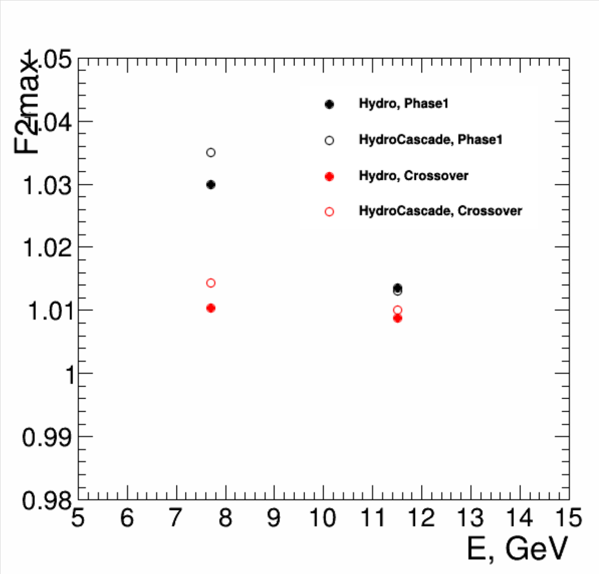
Hydro and HydroCascade separately





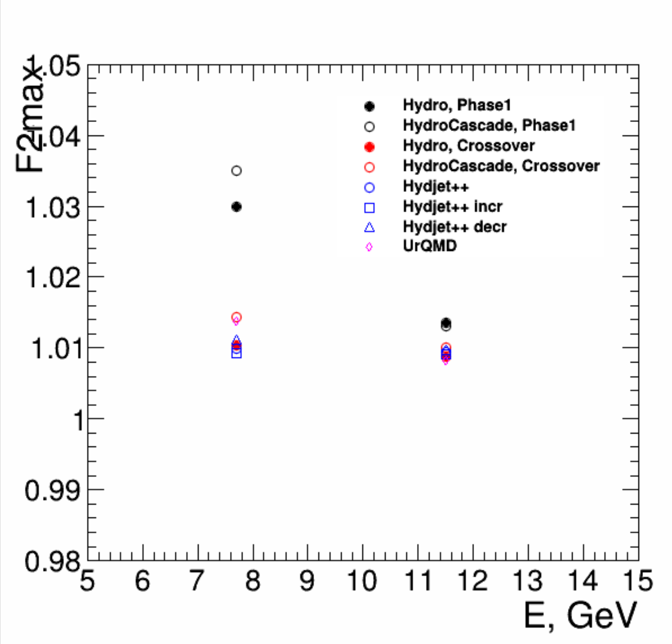






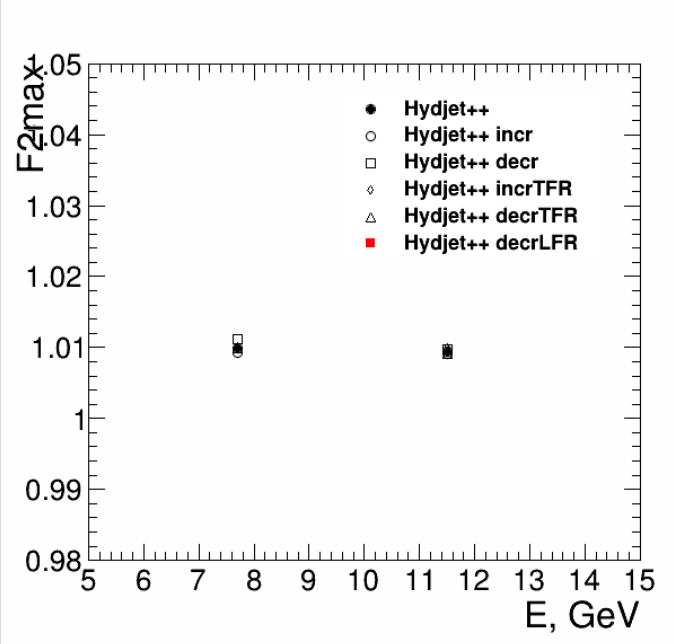
There is a small increase of the *F*2 maximum for HydroCascade. w.r.t Hydro only.
However the different trend in the *F*2 behaviour for the Phase 1 transition and crossover is visible

Models comparison: UrQMD, UrQMD+vHLLE, HYDJET++



UrQMD, HYDJET++ are comparable with vHLLE+UrQMD crossover

Change of multiplicity and volume (transverse and longitudinal) size in HYDJET++
does not affect factorial moments



Summary

- Normalized factorial moments as a function of the size of the observation interval are sensitive to the type of phase transition.
  - We observe the different energy behaviour for the Crossover and 1st order phase transition in the frame of the URQMD+VHLLE model.
  - The energy behaviour is connected to the development of the phase transition itself. Cascade introduces the mild excess to the maximum of the normalized factorial moments.
- We start to work with reconstructed objects. The plan is to take into account the sample efficiency, purity and track momentum resolution.

Factorial moments addition

- Divide interval *D* on *M* bins. *l* – particles are distributed among *m*-groups.
- The probability in the bin (*i*) we will have *n* particles from *m1* group under condition that *l* particles are in *m* groups:

$$R = \frac{\binom{\nu-1}{m1-1}\binom{l-\nu-1}{m-m1-1}}{\binom{l-1}{m-1}}$$

- Under fixed *l*, *m*, *m1*:

$$G_i(l,m,m1) = M \sum_{\nu=m1}^{l-(m-m1)} \frac{\nu * (\nu-1) * \cdots * (\nu-i+1)}{l * (l-1) * \cdots * (l-i+1)} \frac{\binom{\nu-1}{m1-1}\binom{l-\nu-1}{m-m1-1}}{\binom{l-1}{m-1}}$$

Factorial moments addition

- Let's assume the uniform distribution of groups along interval *y*

$$F_i = M^{i-1}\sum_{m=0}^{\infty}\frac{e^{-\lambda}\lambda^m}{m!}\sum_{l=m}^{\infty}\binom{l-1}{m-1}\alpha^{l-m}(1-\alpha)^m\sum_{m1=0}^{m}\binom{m}{m1}\left(\frac{1}{M}\right)^{m1}\left(1-\frac{1}{M}\right)^{m-m1} G_i\,(\mathrm{l,m,m1})$$

- After calculation:

$$F_i(M) = \sum_{k=1}^{i-1} A_k M^{k-1}$$

- For F_2:

$$A_1 = 1 - \frac{2\alpha(1-e^{-\lambda})}{\lambda}$$

$$A_2 = \frac{2\alpha(1-e^{-\lambda})}{\lambda}$$

Resolution of detector