# Correlations of strange particles in relativistic heavy ion collisions

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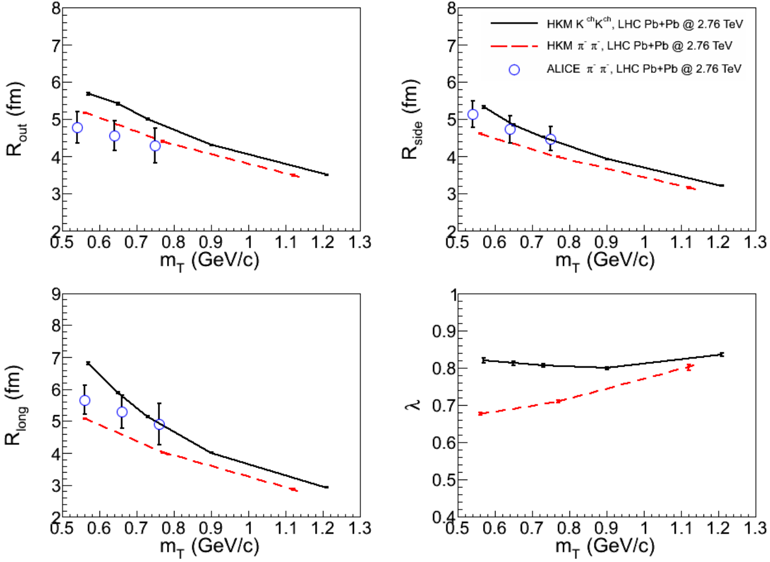
Nantes, July 2017

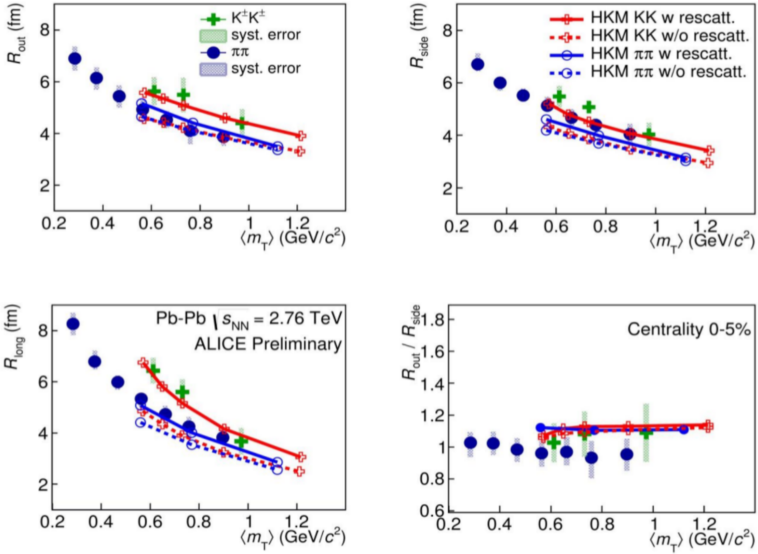
# KK femtoscopy scales at the LHC

*V. M. Shapoval, P. Braun-Munzinger, Iu. A. Karpenko, Yu. M. Sinyukov, Nucl. Phys. A **929** (2014) 1-8*



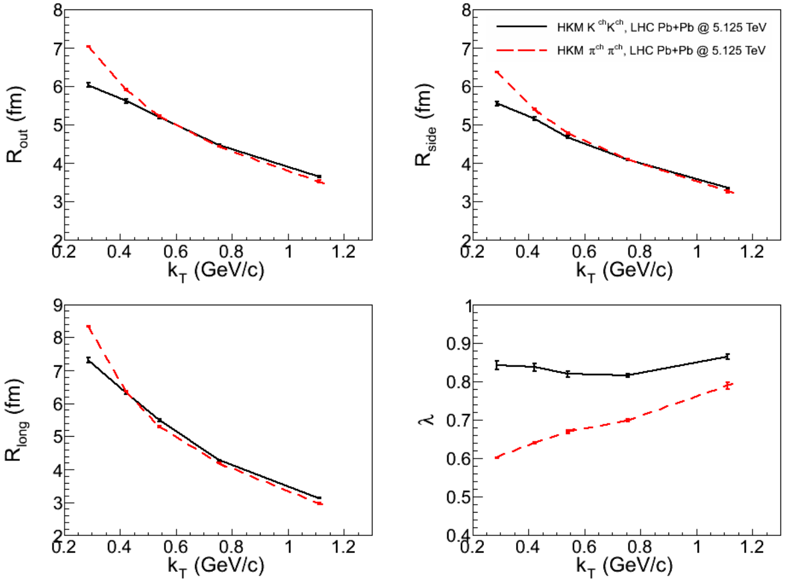
# KK femtoscopy scales at the LHC

*L. V. Malinina, QM2015*



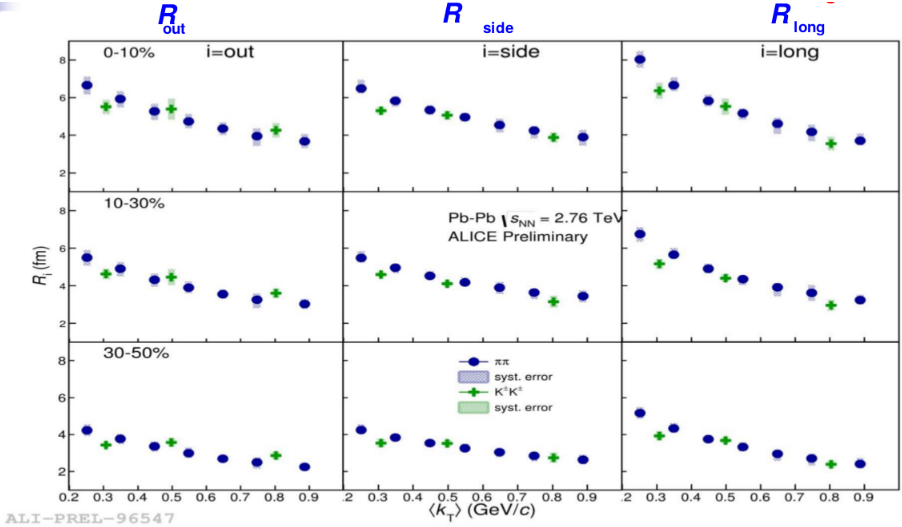
ALI-PREL-96575

# KK femtoscopy scales at the LHC



# KK femtoscopy scales at the LHC

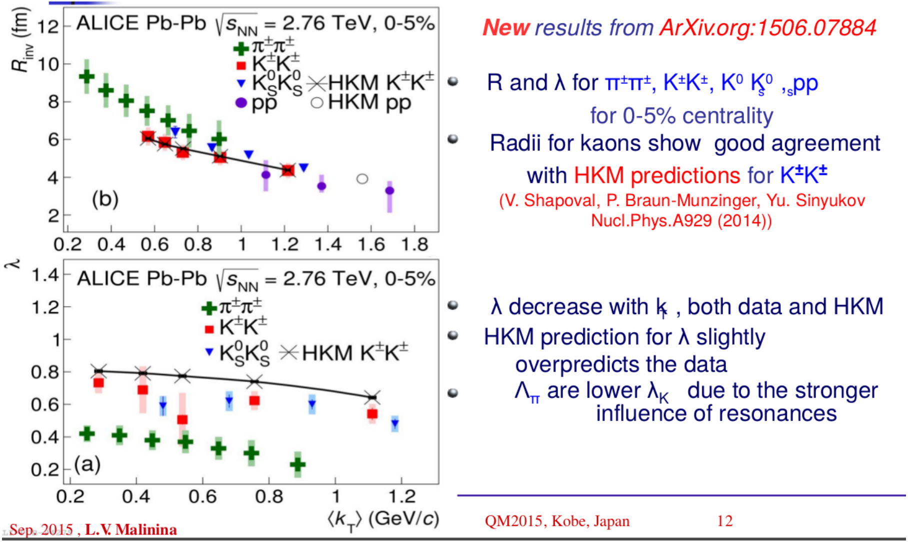
*ALICE Collaboration, arXiv:1507.06842, Phys. Rev. C 93, 024905*



Similar results were reported by PHENIX at RHIC (*PHENIX Collaboration, arXiv:1504.05168, Phys. Rev. C 92, 034914*)

# KK femtoscopy scales at the LHC

*V. M. Shapoval, P. Braun-Munzinger, Iu. A. Karpenko, Yu. M. Sinyukov, Nucl. Phys. A* ***929*** *(2014) 1-8*



***New** results from ArXiv.org:1506.07884*

- R and λ for $\pi^{\pm}\pi^{\pm}$, $K^{\pm}K^{\pm}$, $K^0_s K^0_s$, pp for 0-5% centrality
- Radii for kaons show good agreement with HKM predictions for $K^{\pm}K^{\pm}$ (V. Shapoval, P. Braun-Munzinger, Yu. Sinyukov Nucl.Phys.A929 (2014))
- λ decrease with $k_t$, both data and HKM
- HKM prediction for λ slightly overpredicts the data
- $\Lambda_{\pi}$ are lower $\lambda_K$ due to the stronger influence of resonances

Sep. 2015 , **L.V. Malinina**

QM2015, Kobe, Japan 12

# Space-time picture of $\pi$ and K emission

$$\underbrace{R_l^2(k_T) = \tau^2\lambda^2\left(1+\frac{3}{2}\lambda^2\right)}_{2015} \approx \text{w/o transv. expansion} \approx \underbrace{\tau^2\frac{T}{m_T}}_{1987}\underbrace{\frac{K_2(\frac{m_T}{T})}{K_1(\frac{m_T}{T})}}_{1995}$$

where $\lambda^2 = \frac{T}{m_T}\left(1-\overbrace{\frac{k_T^2}{(m_T+\alpha T)^2}}^{\bar{v}_T^2}\right)^{1/2}$

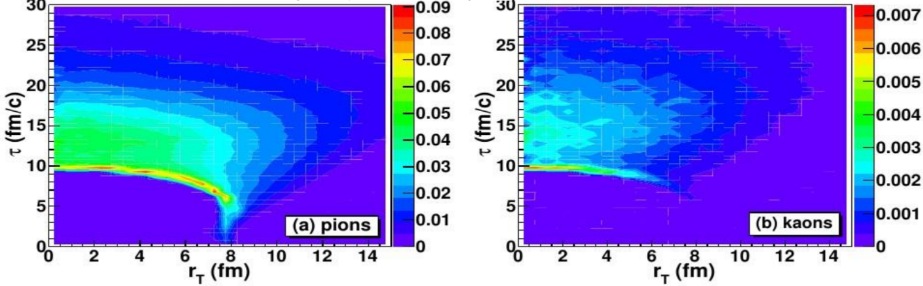
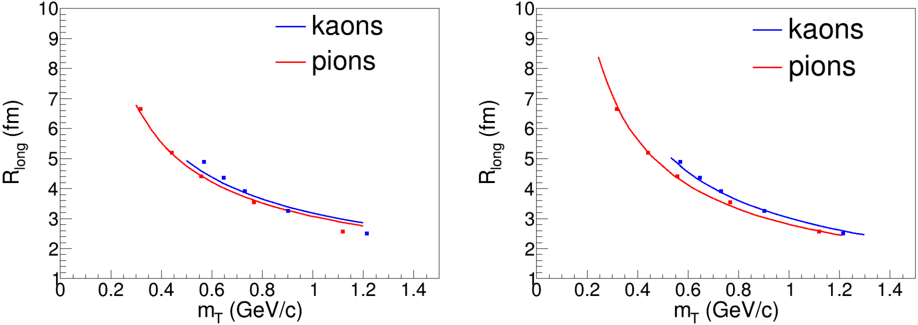


FIG. 4. The momentum angle averaged emission functions per units of space-time and momentum rapidities $g(\tau, r_T, p_T)$ [fm$^{-3}$] (see body text) for pions (a) and kaons (b) obtained from the HKM simulations of Pb+Pb collisions at the LHC $\sqrt{s_{NN}} = 2.76$ GeV, $0.2 < p_T < 0.3$ GeV/$c$, $|y| < 0.5$, $c = 0-5\%$.

From Yu.S., Shapoval, Naboka, Nucl. Phys. A 946 (2016) 247 ( arXiv:1508.01812)

# The collective flow role

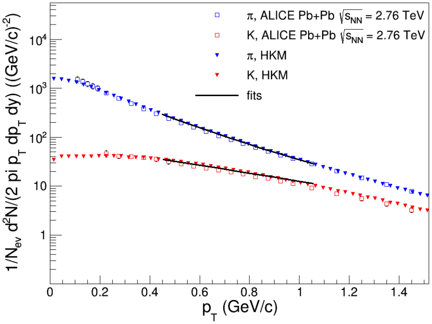
Yu. M. Sinyukov, V. M. Shapoval, V. Yu. Naboka, Nucl. Phys. A 946 (2016), 227

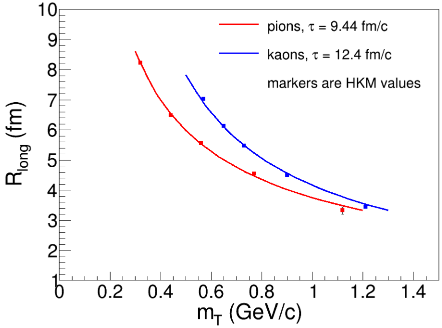


Rescatterings are turned off, $T = 165$ MeV. The flow destroys $m_T$ scaling, since the radii depend on both $m_T/T$ and $k_T/T$.

# Extraction of the maximal emission time

*Yu. M. Sinyukov, V. M. Shapoval, V. Yu. Naboka, Nucl. Phys. A 946 (2016), 227*



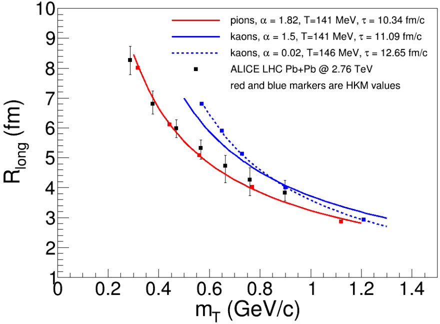


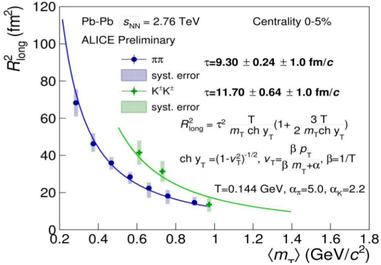
$$p_0 \frac{d^3N}{d^3p} \propto \exp\left[-(m_T/T+\alpha)(1-\bar{v}_T^2)^{1/2}\right]$$

To reduce the effect of the non-Gaussian correlation functions, we take more narrow fitting range for them, $q = 0 - 0.04$ GeV$/c$. The fit parameters $T$ and $\alpha$ correspond to combined pion and kaon spectra fitting. $T_\pi = 147$ MeV, $T_K = 141$ MeV, $\alpha_\pi = 8.5$ and $\alpha_K = 1.5$.

# Extraction of the maximal emission time

*Yu. M. Sinyukov, V. M. Shapoval, V. Yu. Naboka, Nucl. Phys. A 946 (2016), 227*





*L. V. Malinina, QM2015.*

# Lednický & Lyuboshitz analytical model

*V. M. Shapoval, B. Erazmus, R. Lednicky, Yu. M. Sinyukov, arXiv:1405.3594 [nucl-th]*
*R. Lednický, V. L. Lyuboshitz, Yad. Fiz.* ***35****, 1316 (1982).*

$$C(k^*) = \left\langle \left| \Psi^S_{-\mathbf{k}^*}(\mathbf{r}^*) \right|^2 \right\rangle ,$$

where the wave function $\Psi^S$ represents the approximate stationary solution of the scattering problem

$$\Psi^S_{-\mathbf{k}^*}(\mathbf{r}^*) = e^{-i\mathbf{k}^*\cdot\mathbf{r}^*} + \frac{f^S(k^*)}{r^*} e^{ik^*\cdot r^*} .$$

The effective range approximation for the scattering amplitude is utilized

$$f^S(k^*) = \left( \frac{1}{f_0^S} + \frac{1}{2} d_0^S k^{*2} - ik^* \right)^{-1} ,$$

where $f_0^S$ is the **scattering length** and $d_0^S$ is the **effective radius** for a given total spin $S = 1$ or $S = 0$.

The particles are assumed to be **unpolarized** (the polarization $P = 0$) $\Rightarrow$
the fractions of pairs in the singlet and triplet states are $\rho_0 = 1/4(1 - P^2) = 1/4$,
$\rho_1 = 1/4(3 + P^2) = 3/4$.

# Lednický & Lyuboshitz analytical model

The normalized pair separation distribution (**source function**) $S(\mathbf{r}^*) = N^{-1}d^3N/d^3\mathbf{r}^*$ is assumed to be Gaussian

$$S(\mathbf{r}^*) = (2\sqrt{\pi}r_0)^{-3}e^{-\frac{\mathbf{r}^{*2}}{4r_0^2}},$$

where $r_0$ is the effective **source radius**.

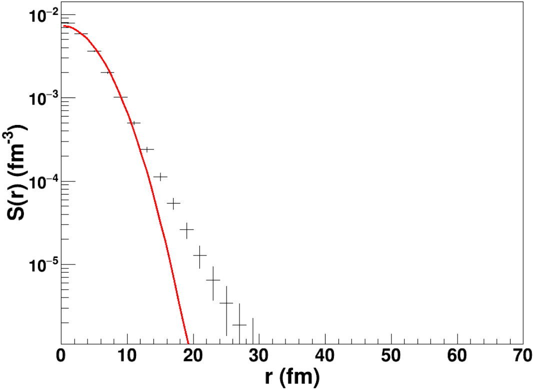
The correlation function can be calculated analytically by averaging $\Psi^S$ over the total spin $S$ and the distribution of the relative distances $S(\mathbf{r}^*)$

$$\begin{aligned} C(k^*) \quad = \quad & 1 + \sum_S \rho_S \left[ \frac{1}{2} \left| \frac{f^S(k^*)}{r_0} \right|^2 \left( 1 - \frac{d_0^S}{2\sqrt{\pi}r_0} \right) + \right. \\ & \left. \frac{2\Re f^S(k^*)}{\sqrt{\pi}r_0} F_1(Qr_0) - \frac{\Im f^S(k^*)}{r_0} F_2(Qr_0) \right], \end{aligned}$$

with $F_1(z) = \int_0^z dx e^{x^2-z^2}/z$ and $F_2(z) = (1-e^{-z^2})/z$.

# $p\Lambda$ angle-averaged source function from HKM

$$S(r) = 1/(4\pi) \int_0^{2\pi} \int_0^{\pi} S(r,\theta,\phi) \sin\theta d\theta d\phi$$



**Source radii from the HKM:**

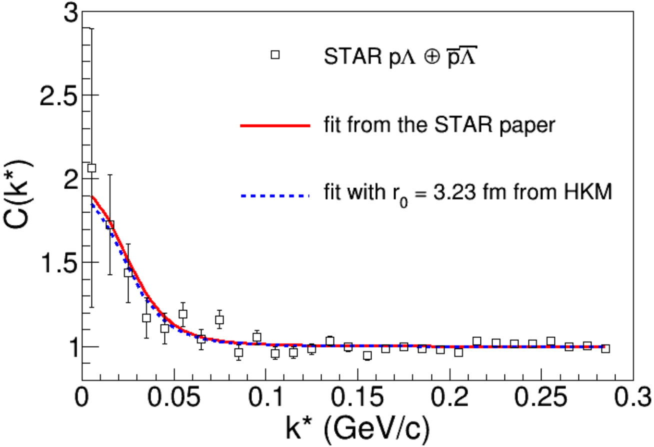
$r_0^{HKM} = 3.23$ fm for $p\Lambda$

$r_0^{HKM} = 3.28$ fm for $p\bar{\Lambda}$

# Baryon-baryon $p\Lambda \oplus \bar{p}\bar{\Lambda}$ correlation function

The **scattering lengths** ($f_0^s = 2.88$ fm, $f_0^t = 1.66$ fm) and **effective radii** ($d_0^s = 2.92$ fm, $d_0^t = 3.78$ fm) for $p - \Lambda$ and $\bar{p} - \bar{\Lambda}$ interaction are taken from
*F. Wang and S. Pratt, Phys. Rev. Lett.* ***83****, 3138 (1999).*

**Source radius from HKM** $r_0^{HKM} = 3.23$ fm.



**Experimental source radius** $r_0^{exp} = 3.09 \pm 0.30^{+0.17}_{-0.25} \pm 0.2$ fm.

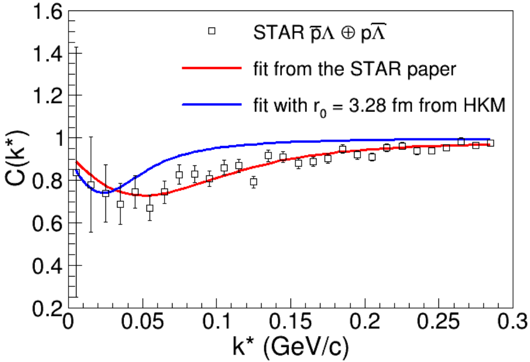
# Baryon-antibaryon $\bar{p}\Lambda \oplus p\bar{\Lambda}$ correlation function

**Assumptions made:**

- $f^s = f^t = f$
- $d_0^s = d_0^t = 0$
- $\Im f_0 > 0$

**Source radius from the HKM:**

$r_0^{HKM} = 3.28$ fm



**Experimental source radius value:**

$r_0^{exp} = 1.50 \pm 0.05^{+0.10}_{-0.12} \pm 0.3$ fm

**Experimental scattering length:**

$\Re f_0 = -2.03 \pm 0.96^{+1.37}_{-0.12}$ fm

$\Im f_0 = 1.01 \pm 0.92^{+2.43}_{-1.11}$ fm

# Account for residual correlations

The data for uncorrected CF are taken from
*G. Renault for the STAR Collaboration, Acta Phys. Hung. A**24**, 131 (2005).*

$$C_{uncorr}(k^*) = 1 + \lambda(k^*)(C(k^*) - 1) + \alpha(k^*)(C_{res}(k^*) - 1), \tag{1}$$

where $\lambda(k^*) = (C_{uncorr}(k^*) - 1)/(C(k^*) - 1)$

$$C_{res}(k^*) = 1 - \tilde{\beta} e^{-4k^{*2}R^2}, \tag{2}$$

Two additional parameters:

- $\tilde{\beta} > 0$ – amplitude of annihilation dip in parent correlations
- $R \ll r_0$ – dip inverse width

Parameters $\Re f_0$, $\Im f_0$, $\tilde{\beta}$, and $R$ are left to vary freely.

**The extracted parameter values are ($\chi^2/\mathrm{ndf} = 0.87$):**

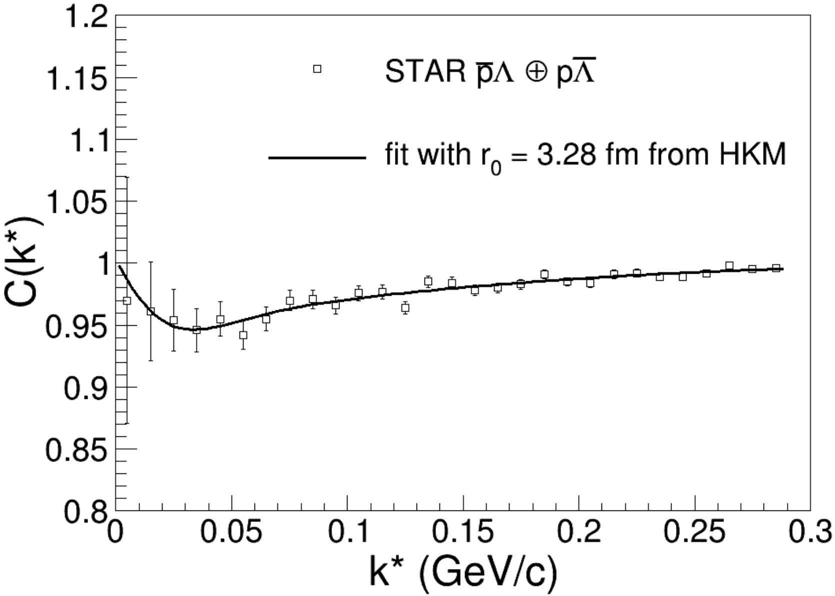
$\Re f_0 = 0.14 \pm 0.66$ fm,

$\Im f_0 = 1.53 \pm 1.31$ fm,

$\tilde{\beta} = 0.034 \pm 0.005$,

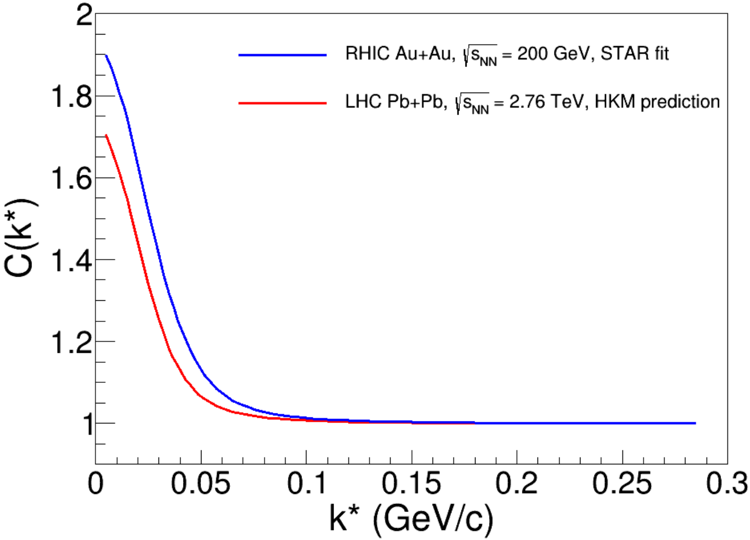
$R = 0.48 \pm 0.05$ fm.

# Account for residual correlations



# Prediction for LHC energy

LHC Pb+Pb collisions at $\sqrt{s_{NN}} = 2.76$ TeV, $c = 0 - 5$ %
$|\eta| < 0.8$, proton $0.7 < p_T < 4.0$ GeV/$c$, Lambda $0.7 < p_T < 5.0$ GeV/$c$.
**HKM radius** $r_0^{HKM} = 3.76$ fm.



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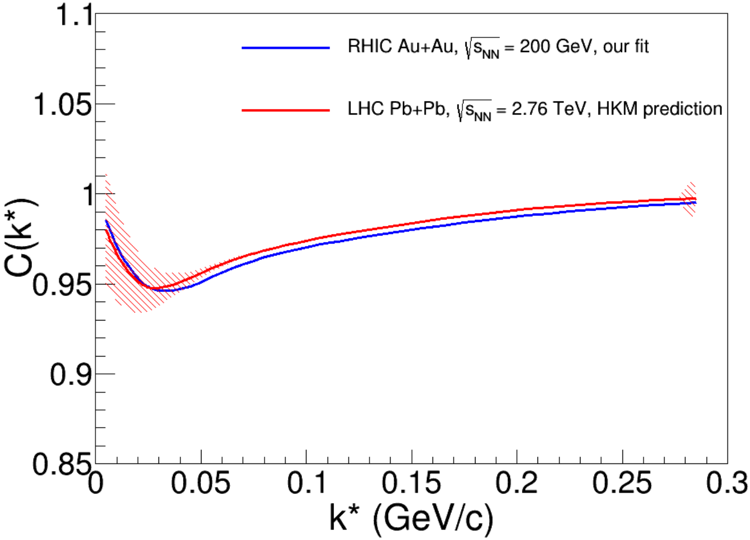


# $p\Xi^-$ case

$$C(\mathbf{q}^*) = \int d^3r^* S(\mathbf{r}^*) |\psi(\mathbf{r}^*, \mathbf{q}^*)|^2. \tag{3}$$

$$\psi_{-\mathbf{k}^*}(\mathbf{r}^*) = \mathrm{e}^{\mathrm{i}\delta_c} \sqrt{A_c(\eta)} \left[ \mathrm{e}^{-\mathrm{i}\mathbf{k}^*\mathbf{r}^*} F(-\mathrm{i}\eta, 1, \mathrm{i}\xi) + f_c(k^*) \frac{\widetilde{G}(\rho, \eta)}{r^*} \right], \tag{4}$$

where $\xi = \mathbf{k}^*\mathbf{r}^* + k^* r^* \equiv \rho(1 + \cos\theta^*)$, $\rho = k^* r^*$, $\eta = (k^* a)^{-1}$, $a = (\mu z_1 z_2 e^2)^{-1}$ two-particle Bohr radius, $\delta_c = \arg\Gamma(1 + \mathrm{i}\eta)$ Coulomb $s$-wave phase shift, $A_c(\eta)$ Coulomb penetration coefficient,

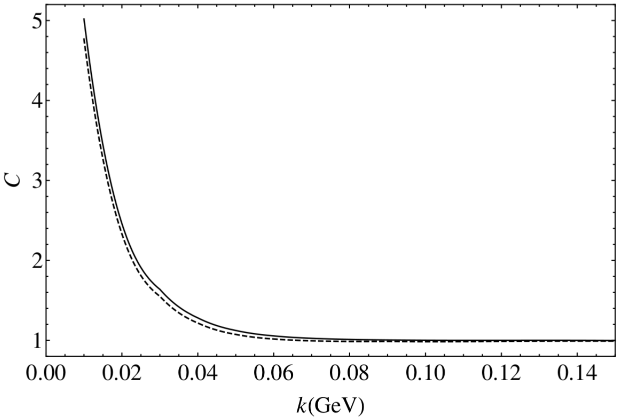
$$F(\alpha, 1, z) = 1 + \alpha z/1!^2 + \alpha(\alpha + 1) z^2/2!^2 + \cdots \tag{5}$$

confluent hypergeometric function, $\widetilde{G} = \sqrt{A_c}(G_0 + \mathrm{i}F_0)$ is a combination of the regular $(F_0)$ and singular $(G_0)$ $s$-wave Coulomb functions:

$$\widetilde{G}(\rho, \eta) = P(\rho, \eta) + 2\eta\rho \left[ \ln|2\eta\rho| + 2C - 1 + \chi(\eta) \right] B(\rho, \eta). \tag{6}$$

Here $C \doteq 0.5772$ – Euler constant, functions $B(\rho, \eta)$, $P(\rho, \eta)$ are defined by recurrence relations.

# $p\Xi^-$ case

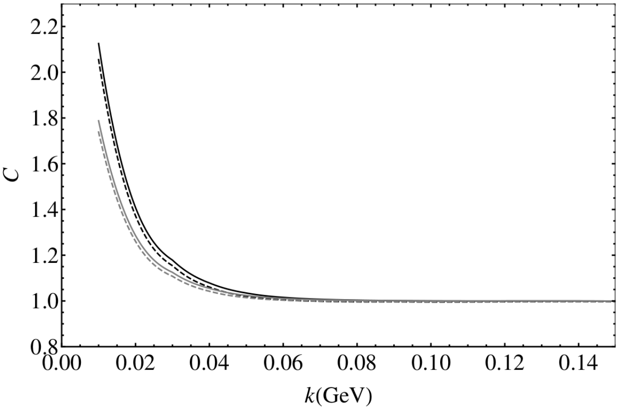


Purity $\lambda = 1$, source radius in iHKM $r_0^{HKM} = 3.1$ fm. Strong interaction scattering length is assumed to be the same as for $p\Lambda$.

Solid line — Coulomb and strong final state interaction exist at all the distances $r$ between baryons in the pair rest frame.

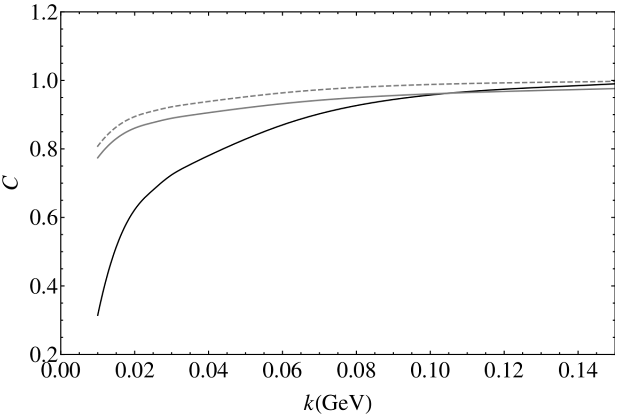
Dashed line — interactions are switched off at $r < 1$ fm.

# $p\Xi^-$ case



Pair purity $\lambda_{res} = 0.28$ (from iHKM). Gray lines: $\lambda = 0.7\lambda_{res} = 0.196$ (possible experimental misidentification is taken into account).

# $p\Xi^+$ case



Source radius $r_0^{iHKM} = 3.1$ fm. Strong interaction scattering length is assumed to be the same as for $p\bar{\Lambda}$, extracted from the STAR data.
Solid line — purity $\lambda = 1$,
Gray dashed line — purity $\lambda_{res} = 0.28$.
Gray solid line — the residual correlations effect is taken into account.

# Conclusions

- $k_T$ scaling instead of $m_T$ one was predicted for kaon and pion femtoscopy radii in HKM, and the prediction was further confirmed by ALICE experimental data.
- The effective characteristics of pion and kaon emission at the LHC were obtained in HKM and a procedure for extracting these parameters from the experimental data was proposed.
- The $p - \Lambda \oplus \bar{p} - \bar{\Lambda}$ and $\bar{p} - \Lambda \oplus p - \bar{\Lambda}$ correlation functions, measured in 10% most central Au+Au collisions by STAR at $\sqrt{s_{NN}} = 200$ GeV, were reproduced using Lednicky and Lyuboshitz analytical formalism with the source radii extracted from the hydrokinetic model (HKM)
- To take into account the residual correlations influencing baryon-antibaryon femtoscopic effects, a modified analytical approximation has been applied. The real and imaginary parts of the spin averaged scattering lenghts have been extracted for baryon-antibaryon pairs.
- Prediction for $p\Lambda$ and $\bar{p}\Lambda$ correlation functions at the LHC energy including residual correlation treatment is done within HKM and Lednicky-Lyuboshitz models.
- Also a prediction for $p\Xi$ correlation functions at the LHC energy are done within iHKM and "Koonin-Pratt" formalism.

Thank you for your attention!

# $p\Xi^-$ pair purity in iHKM

| Pairs | Fractions (%) |
|---|---|
| $p_{prim} - \Xi^-_{prim}$ | 28 |
| $p_{\Lambda} - \Xi^-_{prim}$ | 12 |
| $p_{\Sigma^+} - \Xi^-_{prim}$ | 2 |
| $p_{prim} - \Xi^-_{\Xi(1530)}$ | 38 |
| $p_{\Lambda} - \Xi^-_{\Xi(1530)}$ | 16 |
| $p_{\Sigma^+} - \Xi^-_{\Xi(1530)}$ | 3 |
| $p_{prim} - \Xi^-_{\Omega^-}$ | $< 0.7$ |
| $p_{\Lambda} - \Xi^-_{\Omega^-}$ | $< 0.3$ |
| $p_{\Sigma^+} - \Xi^-_{\Omega^-}$ | $< 0.1$ |