Recent developments in EPOS concerning parton saturation

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EPOS: Based on multiple scattering and flow

Several steps (even in pp!):

1) Initial conditions:
Gribov-Regge **multiple scattering** approach,
elementary object = Pomeron = parton ladder,
Nonlinear effects via saturation scale Q_s

2) Core-corona approach
to separate fluid and jet hadrons

3) Viscous **hydrodynamic expansion**, $\eta/s = 0.08$

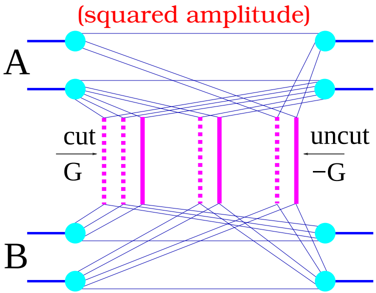
4) Statistical hadronization, final state hadronic cascade

arXiv:1312.1233 , arXix:1307.4379

Initial conditions: Marriage pQCD+GRT+energy sharing

(Drescher, Hladik, Ostapchenko, Pierog, and Werner, Phys. Rept. 350, 2001)

For pp, pA, AA:



$$\sigma^{\text{tot}} = \sum_{\text{cut P}} \int \underbrace{\sum_{\text{uncut P}} \int}_{d\sigma_{\text{exclusive}}}$$

$$\text{cut Pom}: G = \frac{1}{2\hat{s}} 2\text{Im}\left\{\mathcal{FT}\{T\}\right\}(\hat{s}, b),\; T = i\hat{s}\,\sigma_{hard}(\hat{s})\,\exp(R_{\text{hard}}^2 t)$$

Nonlinear effects considered via saturation scale Q_s

Some facts about the Gribov-Regge multiple scattering scheme (the heart of the EPOS approach)

S-matrix:

$$|\psi(t=+\infty)\rangle = \hat{S}\,|\psi(t=-\infty)\rangle$$

Unitarity relation:

$$\hat{S}^{\dagger}\hat{S} = 1$$

which leads to ($\sum$ includes phase space integration)

$$\underbrace{\sum_f (2\pi)^4 \delta(p_f - p_i)\, |T_{fi}|^2}_{2s\,\sigma_{\mathrm{tot}}} = \frac{1}{i}(T_{ii} - T_{ii}^*)$$

$$= 2\mathrm{Im} T_{ii}$$

$$= \frac{1}{\mathrm{i}} \mathrm{disc}\, T_{ii}$$

with (s, t : Mandelstam variables)

$$\mathrm{disc}\, T_{ii} = T_{ii}(s + i\epsilon, t) - T_{ii}(s - i\epsilon, t)$$

In detail :

$$\begin{aligned} 1 &= \langle i|\,\hat{S}^{\dagger}\hat{S}\,|i\rangle \\ &= \sum_f \langle i|\,\hat{S}^{\dagger}\,|f\rangle\,\langle f|\,\hat{S}\,|i\rangle \\ &= \sum_f \langle f|\,\hat{S}\,|i\rangle^{*}\,\langle f|\,\hat{S}\,|i\rangle \end{aligned}$$

So

$$1 = \sum_f S^{*}_{fi} S_{fi}$$

Using $S_{fi} = \delta_{fi} + i(2\pi)^4\delta(p_f - p_i)T_{fi}$, dividing by $i(2\pi)^4\delta(0)$

$$\begin{aligned} \frac{1}{i}\left(T_{ii} - T^{*}_{ii}\right) &= \sum_f (2\pi)^4\delta(p_f - p_i)\,|T_{fi}|^2 \\ &= 2w\,\sigma_{\mathrm{tot}} \\ &= 2s\sigma_{\mathrm{tot}} \end{aligned}$$

The l.h.s. :

$$\frac{1}{i}\left(T_{ii} - T^{*}_{ii}\right) = 2\mathrm{Im}T_{ii}$$

So we get the optical theorem

$$2\mathrm{Im}T_{ii} = \sum_f (2\pi)^4 \delta(p_f - p_i)\, |T_{fi}|^2 = 2s\,\sigma_{\mathrm{tot}}$$

Assume:

- T_{ii} is Lorentz invariant $\rightarrow$ use s, t
- $T_{ii}(s,t)$ is an analytic function of s, with s considered as a complex variable (Hermitean analyticity)
- $T_{ii}(s,t)$ is real on some part of the real axis (see optical theorem)

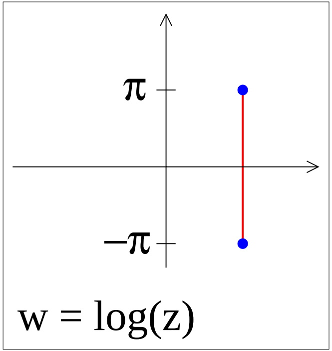
Using the Schwarz reflection principle, $T_{ii}(s,t)$ first defined for $\mathrm{Im}s \geq 0$ can be continued in a unique fashion via $T_{ii}(s^*,t) = T_{ii}(s,t)^*$. So:

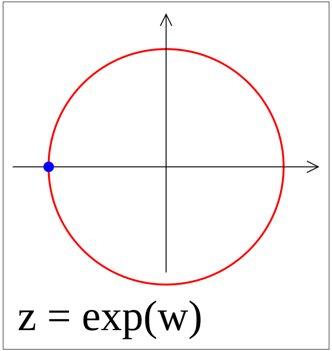
$$\frac{1}{i}\left(T_{ii}(s,t) - T_{ii}(s,t)^*\right) = \frac{1}{i}\left(T_{ii}(s,t) - T_{ii}(s^*,t)\right) = \frac{1}{i}\mathrm{disc}\,T_{ii}$$

with

$$\mathrm{disc}\,T_{ii} = T_{ii}(s+i\epsilon,t) - T_{ii}(s-i\epsilon,t).$$

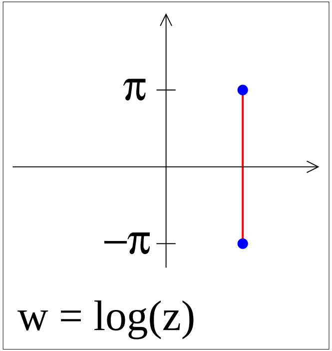
Discontinuity, example: exp and its inverse log

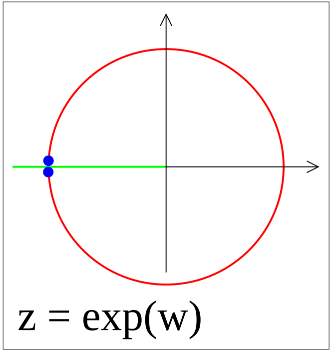




Problem: exp(w) maps two points to one,
inversion not possible

unless one excludes the green line





and shifts points on this line up or down (by ϵ).
Discontinuity = $2\pi i$

In detail: Let us study the mapping

$$w \rightarrow z \text{ with } z = \exp(w).$$

Consider $w = x + iy$, with x fixed and y going from $-\pi$ to π, which means we have a trajectory going from $w_1 = x - i\pi$ to $w_2 = x + i\pi$.
Doing the mapping $w \rightarrow z$ for this trajectory, we get

$$z = e^x e^{iy}$$

describing a circular trajectory with radius e^x with start and end point $z_1 = z_2 = -e^x$

Doing the inverse mapping $z \rightarrow w = \log(z)$: We get for $z_1 = z_2$ two different values w_1 and w_2 !!

One has to define log in $\mathbb{C} - \mathbb{R}_{\leq 0}$ (branch). The negative real axis is called branch cut.
The discontinuity at $z = -e^x$:

$$\log(z + i\epsilon) - \log(z - i\epsilon) = 2\pi i$$

Back to our T-matrix : We have

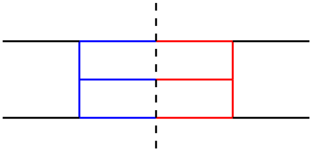
$$2s\,\sigma_{\mathrm{tot}} = \sum_f (2\pi)^4 \delta(p_f - p_i)\,|T_{fi}|^2 = \frac{1}{\mathrm{i}}\mathrm{disc}\,T_{ii}$$

$\frac{1}{\mathrm{i}}\mathrm{disc}\,\boldsymbol{T}$ can be seen as a so-called “cut diagram”, with modified Feynman rules, the “intermediate particles” are on mass shell, and we use simply

$$2s\,\sigma_{\mathrm{tot}} = \frac{1}{\mathrm{i}}\mathrm{disc}\,T_{ii}$$

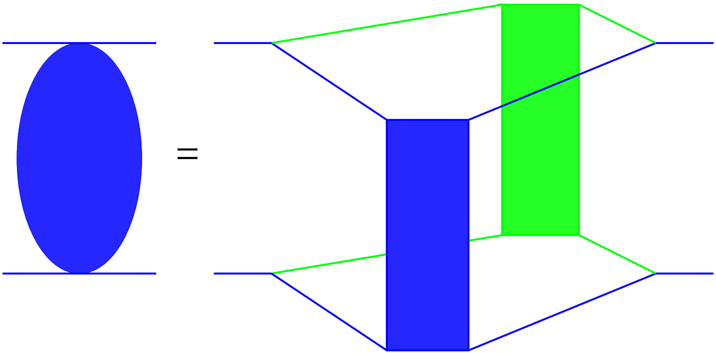
Modified Feynman rules :

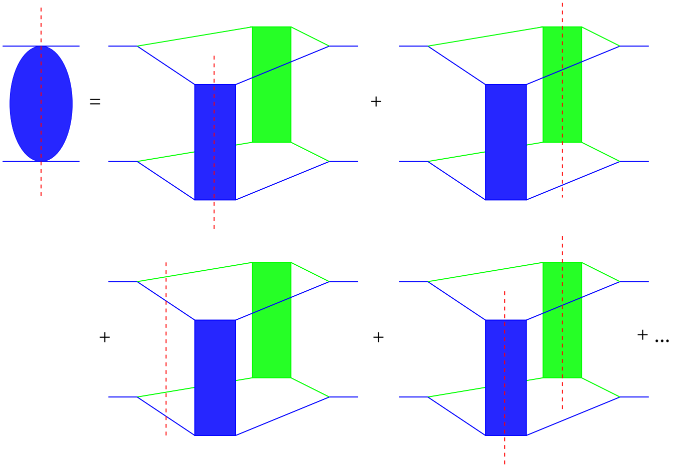
- □ Draw a dashed line from top to bottom



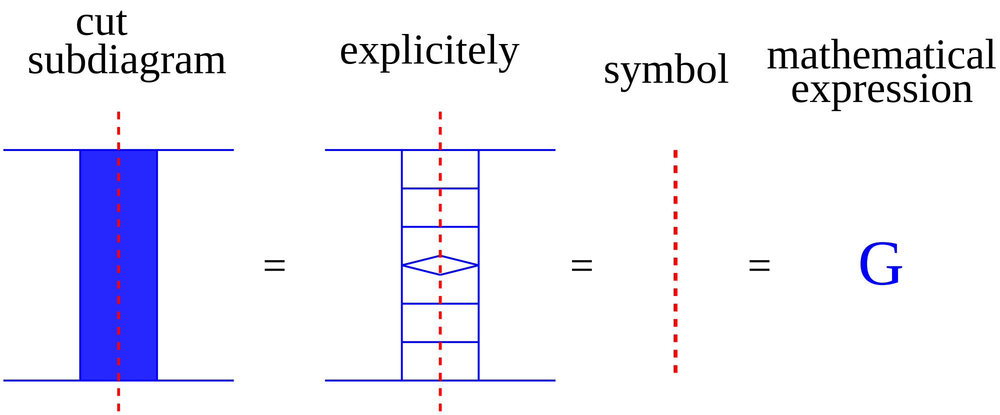
- □ Use “normal” Feynman rules to the left
- □ Use the complex conjugate expressions to the right
- □ For lines crossing the cut: Replace propagators by mass shell conditions $2\pi\theta(p^0)\delta(p^2 - m^2)$

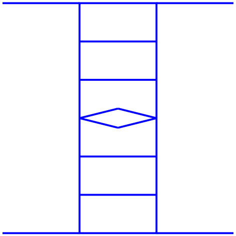
Useful in case of substructures:





Cut diagram
= sum of products of cut/uncut subdiagrams


cut subdiagram
explicitely
symbol
mathematical expression
=
=
=
G

uncut subdiagram		explicitely		symbol		mathematical expression
	=		=		=	$-G$

Single Pomeron contribution G, computed via pQCD, can be (very well) fitted as*)

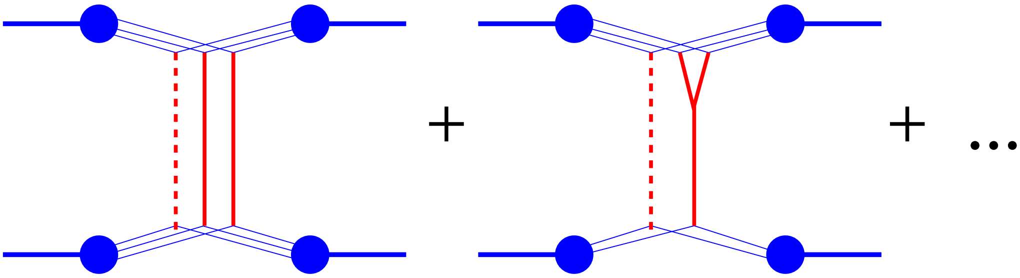
$$G \approx G_{\text{fit}} = \alpha\,(x^{+})^{\beta}(x^{-})^{\beta'}$$

($x^{\pm}$are light cone momentum fractions)

Extremely useful! Allows analytical calculations of cross sections.

*) (Drescher, Hladik, Ostapchenko, Pierog, and Werner, Phys. Rept. 350, 2001)

Consistency requires adding more diagrams (ladder splitting/fusion, triple Pomeron vertices, gluon fusion in CGC ...)



(Drescher, Hladik, Ostapchenko, Pierog, and Werner, Phys. Rept. 350, 2001)

Motivated by model calculations, we treat ladder fusion via adding an exponent [1] :

$$\boldsymbol{G_{\mathrm{fit}} \longrightarrow G_{\mathrm{eff}} = \alpha\,(x^{+})^{\beta+\varepsilon^{\mathrm{proj}}}(x^{-})^{\beta'+\varepsilon^{\mathrm{targ}}}}$$

("epsilon method") with

$$\varepsilon = \varepsilon(Z),$$

depending on "the number of participants":

$$Z^{\mathrm{proj}} = \sum_{\mathrm{proj\,nucleons\,i'}} f_{\mathrm{part}}\left(|\vec{b} + \vec{b_{i'}} - \vec{b_j}|\right)$$

(j is the target nucleon the Pomeron is connected to)

[1] K.Werner, FM.Liu, T.Pierog, Phys.Rev. C74 (2006) 044902

Advantages

- Cross section calculations perfectly doable
- Energy dependence of $\sigma_{\rm tot}$, $\sigma_{\rm el}$ (and more) correct

Big problems

- **Adding ε does not change the internal Pomeron structure**
- No binary scaling in pA at high p_t (tails much too low)

Solution

- Introducing a **saturation scale**
  (K. Werner, B. Guiot, Iu. Karpenko, T. Pierog, Phys.Rev. C89 (2014) 064903)

Before: Compute G with fixed soft cutoff Q_0
$\rightarrow$ fit $\rightarrow$ add ε exponents

New: Compute G with saturation scale $Q_s \propto Z\,\hat{s}^{\lambda}$
$\rightarrow$ fit (${\hat{s}}$ = Pomeron invariant mass)

varying Q_s changes internal structure!

Still something missing ...

□ The saturation scale depends on the number of **participating nucleons**,

□ but NOT on the **number of Pomerons** N_{Pom} (participating parton pairs)

The number of Pomerons represents the event activity in pp, as the number of participating nucleons does in pA.

The final solution

- ☐ Combining “epsilon method” and saturation scale in a smart way (T. Pierog and K. Werner, procs. EDS 2015, Borgo, France)

Step 1 Compute $G = G(Q_0)$ with fixed soft cutoff Q_0 $\rightarrow$ fit $\rightarrow$ add ε exponents ($\rightarrow$ G_{eff}) in order to fit cross sections

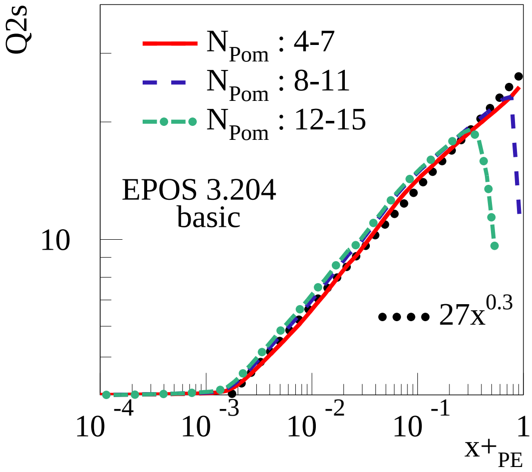
Step 2 Introduce saturation scale via

$$G_{\mathrm{eff}} = k\, G(Q_s)$$

affecting the internal structure

(We will see what to take to k)

The saturation scale Q_s^2



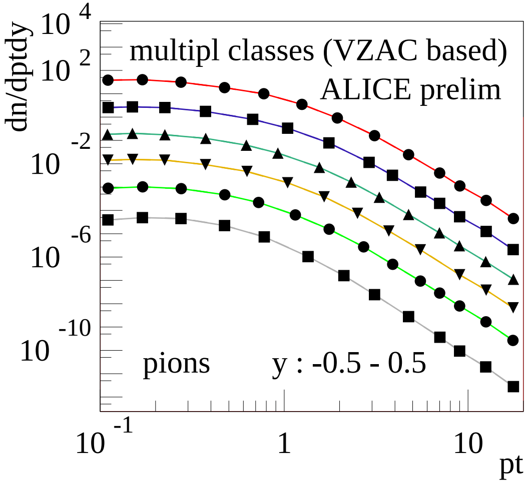
pp at 7 TeV

using $G_{\rm eff} = k\,G(Q_s)$

with constant k

($x+_{\rm PE}$ is the LC momentum fraction on the projectile side)

A crucial test:
Multiplicity dependence of spectra at high p_t



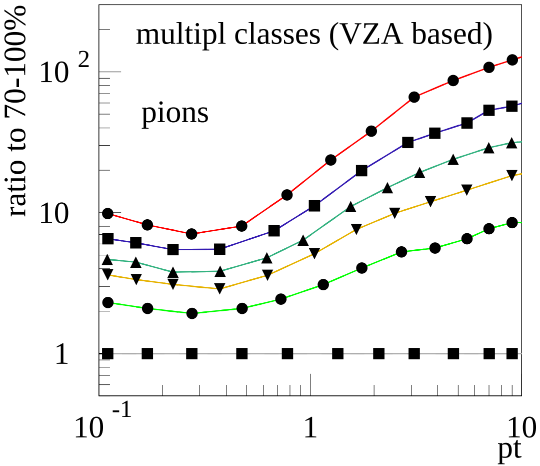
preliminary ALICE data

(digitalized from B.A.Hess, talk at MPI@LHC 2015 Trieste November 27, 2015)

multiplicity bins (top to bottom): 0-1%, 1-5%, 10-15%, 20-30%, 40-50%, 70-100%

lines to guide the eye

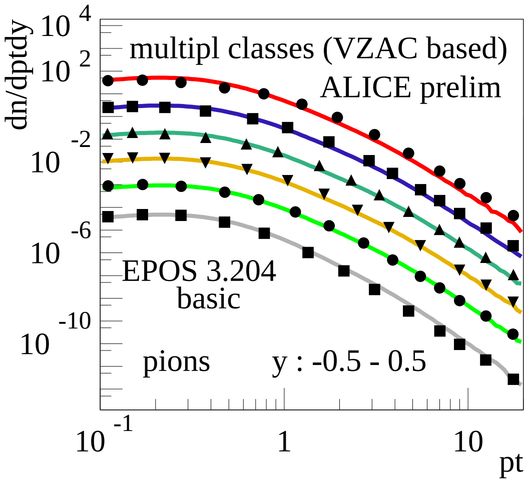
Same data - ratio to 70-100%



non-trivial:

spectra get harder with multiplicity

Comparing ALICE data with EPOS calculations



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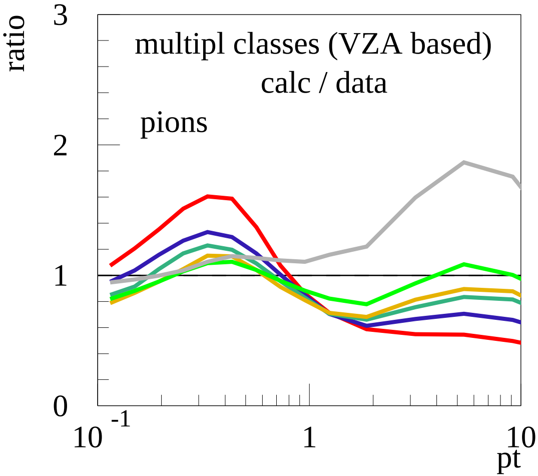
multiplicity bins
(top to bottom):
0-1%, 1-5%, 10-15%, 20-30%, 40-50%, 70-100%

Not too bad for a first shot ... but tails are not correct

Comparing ALICE data with EPOS calculations

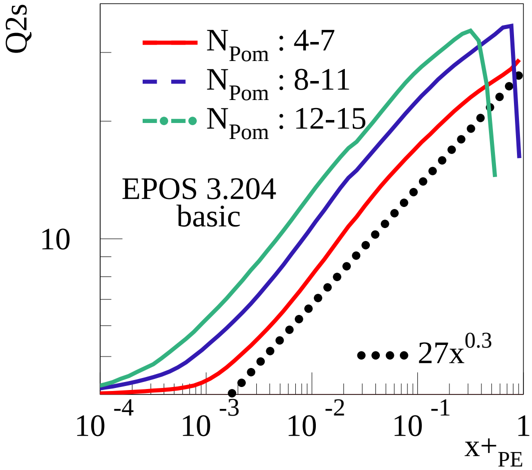
Ratio calculation / data



multiplicity bins : 0-1% (red) , 1-5%, 10-15%, 20-30%, 40-50%, 70-100% (grey)

Tails wrong by factors of two (low pt will be modified by hydro)

Make saturation scale Q_s^2 depending on N_{Pom}



pp at 7 TeV

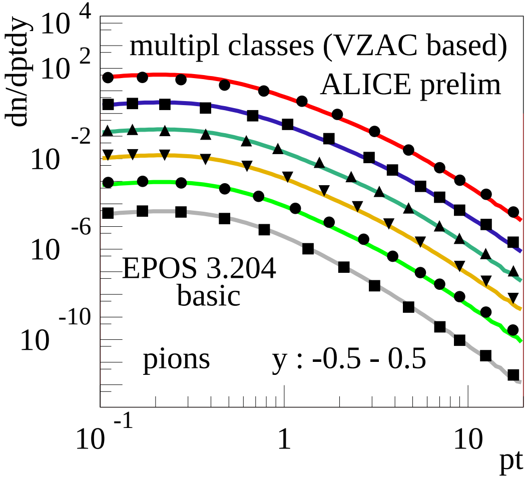
using $G_{\mathrm{eff}} = k\, G(Q_s)$

with

$$k = \left(\frac{N_{\mathrm{Pom}}}{\langle N_{\mathrm{Pom}}\rangle}\right)^{0.75}$$

higher Q_s^2 with increasing Pomeron number (like N_{part} dependence in pA)

Comparing ALICE data with EPOS calculations

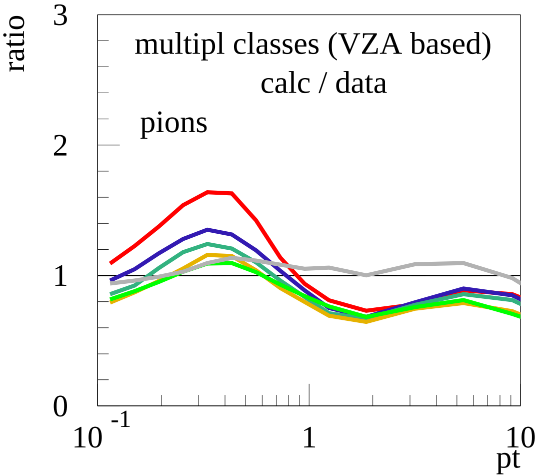


using

$$k = \left(\frac{N_{\mathrm{Pom}}}{\langle N_{\mathrm{Pom}}\rangle}\right)^{0.75}$$

=> much better

Comparing ALICE data with EPOS calculations
Ratio calculation / data



using

$$k = \left(\frac{N_{\mathrm{Pom}}}{\langle N_{\mathrm{Pom}} \rangle}\right)^{0.75}$$

multiplicity bins : 0-1% (red) , 1-5%, 10-15%, 20-30%, 40-50%, 70-100% (grey)

Tails reasonable (low pt will be modified by hydro**)**

Summary

- **New (and final?) major improvement of the multiple scattering scheme in EPOS: Pomeron number dependence of the saturation scale** (**and the corresponding technical improvements which make it possible**)
- **Provides increasing Pomeron hardness with increasing multiplicity** (**ALICE multipl dependence of spectra**)
- **Explains strong increase of high pt charm production vs multiplicity** (**not discussed here**)