Correlation measurements of particle interaction at STAR

R. Lednický @ JINR Dubna & IP ASCR Prague

- History
- QS correlations $\rightarrow$ femtoscopy with identical particles
- FSI correlations $\rightarrow$ femtoscopy with nonidentical particles
- Correlation study of strong interaction
- Summary

History of Correlation femtoscopy

measurement of space-time characteristics R, $c\tau$ ~ fm
of particle production using particle correlations

Fermi'34, GGLP'60, Dubna (GKPLL..'71-) …

β-decay: Coulomb FSI between $e^{\pm}$ and Nucleus in β-decay modifies the relative momentum (**k**) distribution → Fermi (correlation) function

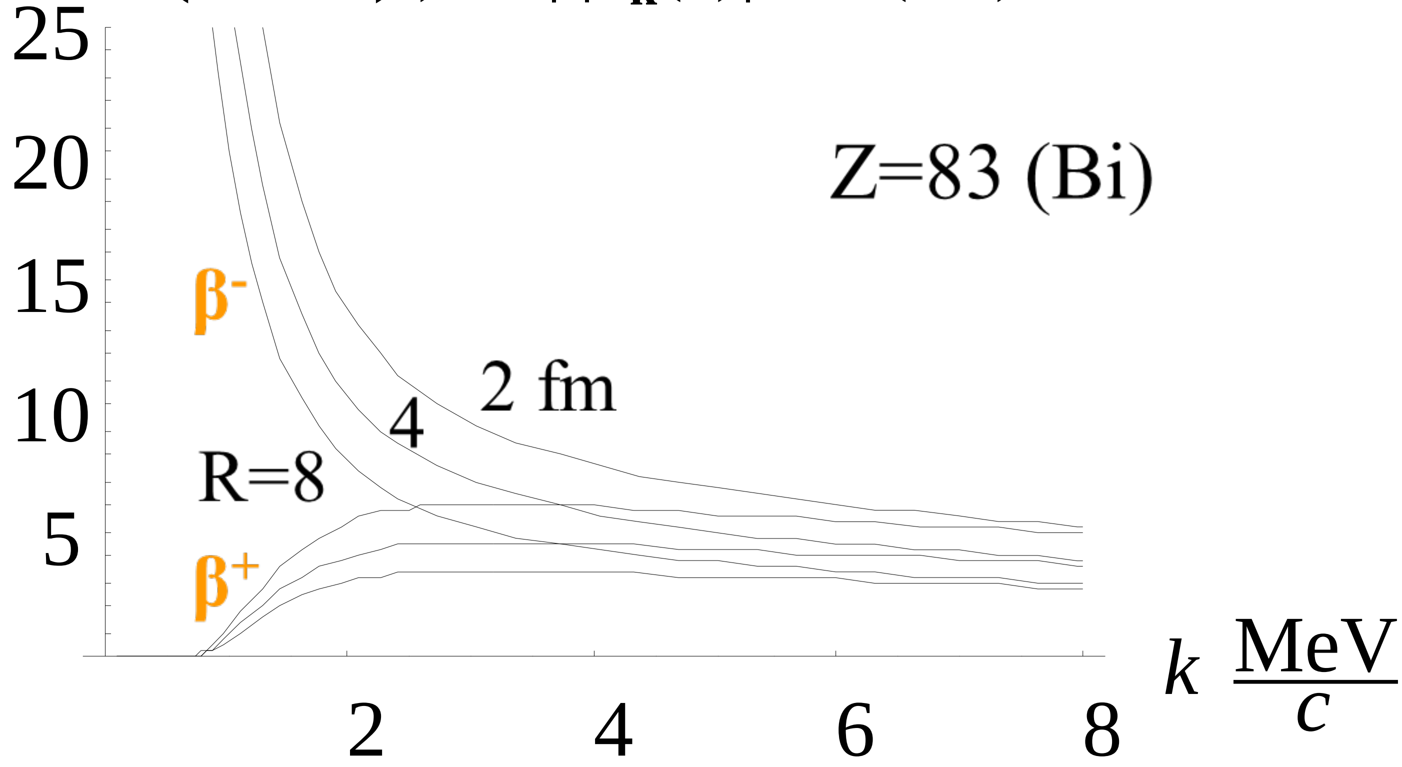
$$F(k,Z,R)= \langle|\psi_{-\mathbf{k}}(\mathbf{r})|^2\rangle$$

is sensitive to Nucleus radius R if charge Z » 1

$\psi_{-\mathbf{k}}(\mathbf{r})$ = electron – Nucleus WF ($\Delta t=0$)

Fermi function in β-decay

$$F(k,Z,R)\ ) = \langle|\psi_{-\mathbf{k}}(\mathbf{r})|^2\rangle \sim (kR)^{-(Z/137)^2}$$



Modern correlation femtoscopy formulated by Kopylov & Podgoretsky

KP'71-75: **settled basics of correlation femtoscopy in > 20 papers (for non-interacting identical particles)**

- **proposed CF= N^{corr}/N^{uncorr} & mixing techniques to construct N^{uncorr} & two-body approximation to calculate theor. CF**
- **showed that sufficiently smooth momentum spectrum allows one to neglect space-time coherence at small q* smoothness approximation:**

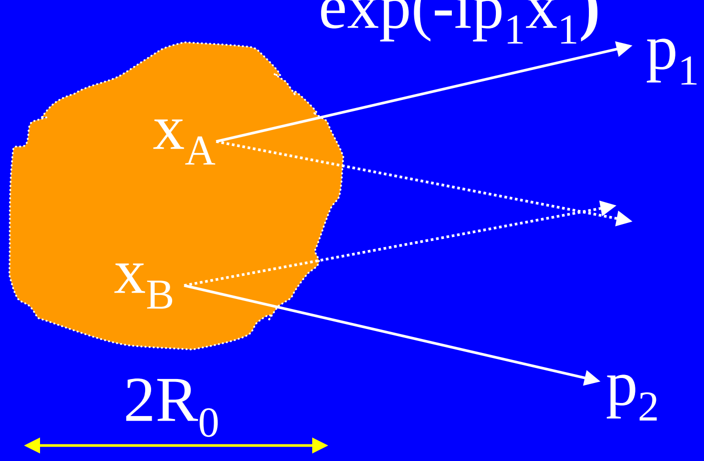
$$|\int d^4x_1 d^4x_2 \psi_{p1p2}(x_1,x_2)...|^2 \rightarrow \int d^4x_1 d^4x_2 |\psi_{p1p2}(x_1,x_2)|^2...$$

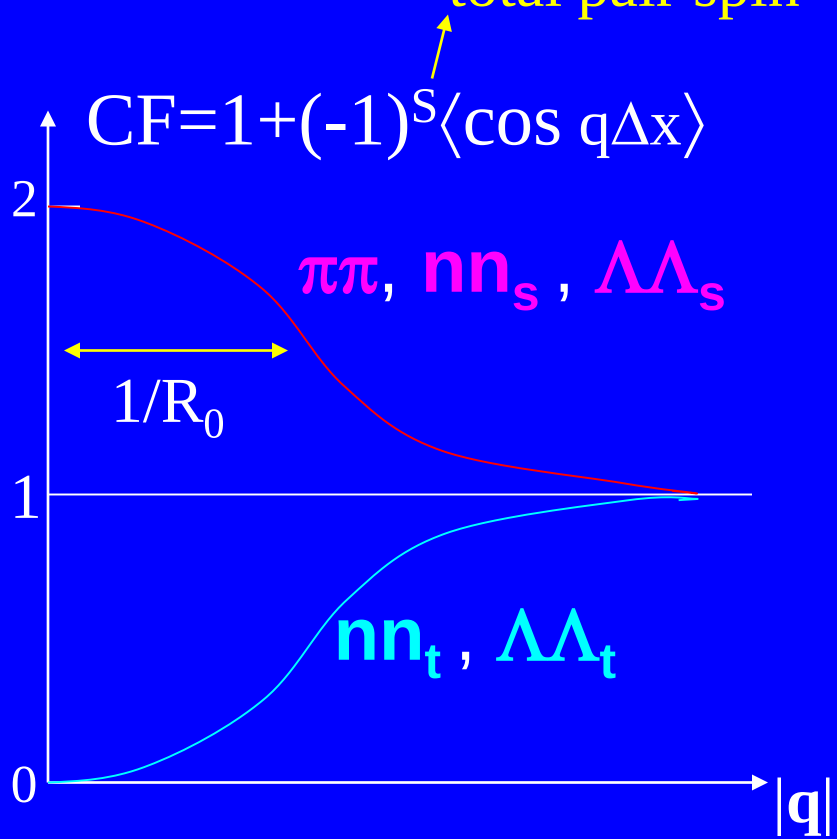
- **clarified role of space-time production characteristics: shape & time source picture from various q-projections**

QS symmetrization of production amplitude

→ *momentum correlations of identical particles are sensitive to space-time structure of the source*

KP'71-75





$q = p_1 - p_2 \xrightarrow{PRF} \{0, 2\mathbf{k^*}\}$

$\Delta x = x_A - x_B \rightarrow \{t^*, \mathbf{r^*}\}$

$$CF \rightarrow \langle\, |\psi^{S(sym)}_{-k^*}(r^*)|^2 \,\rangle = \langle\, |\, [\, e^{-ik^*r^*} + (-1)^S e^{ik^*r^*}]/\sqrt{2}\, |^2 \,\rangle$$

! CF of noninteracting identical particles is independent of t* in PRF

KP model of single-particle emitters

Probability amplitude to observe a particle with 4-coordinate x from emitter A at x_A can depend on $x-x_A$ only and so can be written as:

$$\langle x|\psi_A\rangle = (2\pi)^{-4}\int d^4\kappa\, u_A(\kappa)\exp[i\kappa(x-x_A)].$$

Transfering to 4-momentum representation: $\langle p|x\rangle = \exp(-ipx)$ $\Rightarrow$

$$\langle p|\psi_A\rangle = \int d^4x\, \langle p|x\rangle\langle x|\psi_A\rangle = u_A(p)\exp(-ipx_A)$$

and probability amplitude to observe two spin-0 bosons:

$$T^{\mathrm{sym}}_{AB}(p_1,p_2) = [\langle p_1|\psi_A\rangle\langle p_2|\psi_B\rangle + \langle p_2|\psi_A\rangle\langle p_1|\psi_B\rangle]/\sqrt{2}.$$

Corresponding **momentum correlation function**:

$\Delta x = x_A - x_B$

$$R(p_1,p_2) = 1 + \frac{\Re\sum_{AB} u_A(p_1)u_B(p_2)u^*_A(p_2)u^*_B(p_1)\exp(-iq\Delta x)}{\sum_{AB}|u_A(p_1)u_B(p_2)|^2} \doteq 1 + \langle\cos(q\Delta x)\rangle$$

if $u_A(p_1) \approx u_A(p_2)$: "smoothness assumption"

Assumptions to derive KP formula

$$\mathrm{CF} - 1 \propto \langle \cos q\Delta x \rangle$$

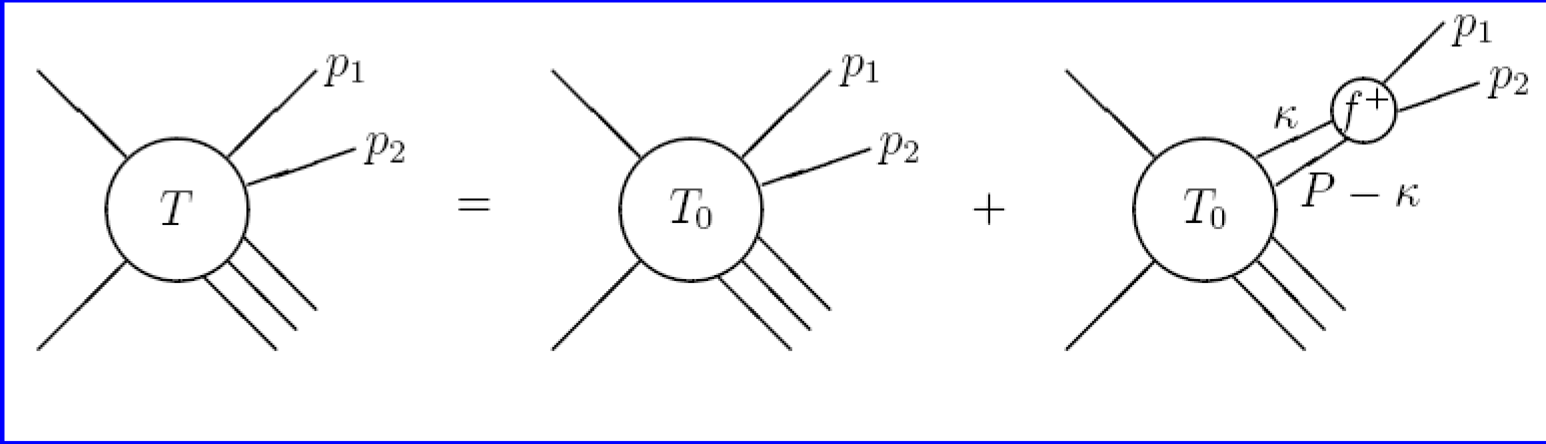
- **two-particle approximation (small freeze-out PS density f)**
  ~ OK, $\langle f \rangle \ll 1$? low p_t

- **smoothness approximation: $R_{emitter} \ll R_{source} \Leftrightarrow \langle|\Delta p|\rangle \gg \langle|q|\rangle_{peak}$**
  ~ OK in HIC, $R_{source}^2 \gg 0.1\ \mathrm{fm}^2 \approx p_t^2$-slope of direct particles

- **neglect of FSI**
  OK for photons, ~ OK for pions up to Coulomb repulsion

- **incoherent or independent emission**
  2π and 3π CF data approx. consistent with KP formulae:
  $\mathrm{CF}_3(123) = 1+|F(12)|^2+|F(23)|^2+|F(31)|^2+2\mathrm{Re}[F(12)F(23)F(31)]$
  $\mathrm{CF}_2(12) = 1+|F(12)|^2$, $F(q)| = \langle e^{iqx} \rangle$

FSI: plane waves → BS-amplitude Ψ



$$T(p_1,p_2;\alpha) = T_0(p_1,p_2;\alpha) + \Delta T(p_1,p_2;\alpha)$$

$$\Delta T(p_1,p_2;\alpha) = \frac{\mathrm{i}\sqrt{P^2}}{2\pi^3}\int \mathrm{d}^4\kappa \frac{T_0(\kappa, P-\kappa;\alpha) f^{S*}(p_1,p_2;\kappa,P-\kappa)}{(\kappa^2 - m_1{}^2 - \mathrm{i}0)[(P-\kappa)^2 - m_2{}^2 - \mathrm{i}0]}$$

Inserting KP amplitude $T_0(p_1,p_2;\alpha) = u_A(p_1)u_B(p_2)\exp(-ip_1x_A - ip_2x_B)$ in ΔT and taking the amplitudes $u_A(\kappa)$ and $u_B(P-\kappa)$ out of the integral *at* $\kappa \approx p_1$ and $P-\kappa \approx p_2$ (again "smoothness assumption") ⇒

Product of plane waves → BS-amplitude Ψ :

$$T(p_1,p_2;\alpha) = u_A(p_1)u_B(p_2)\,\Psi_{p_1p_2}(x_A, x_B)$$

Effect of nonequal times in pair cms

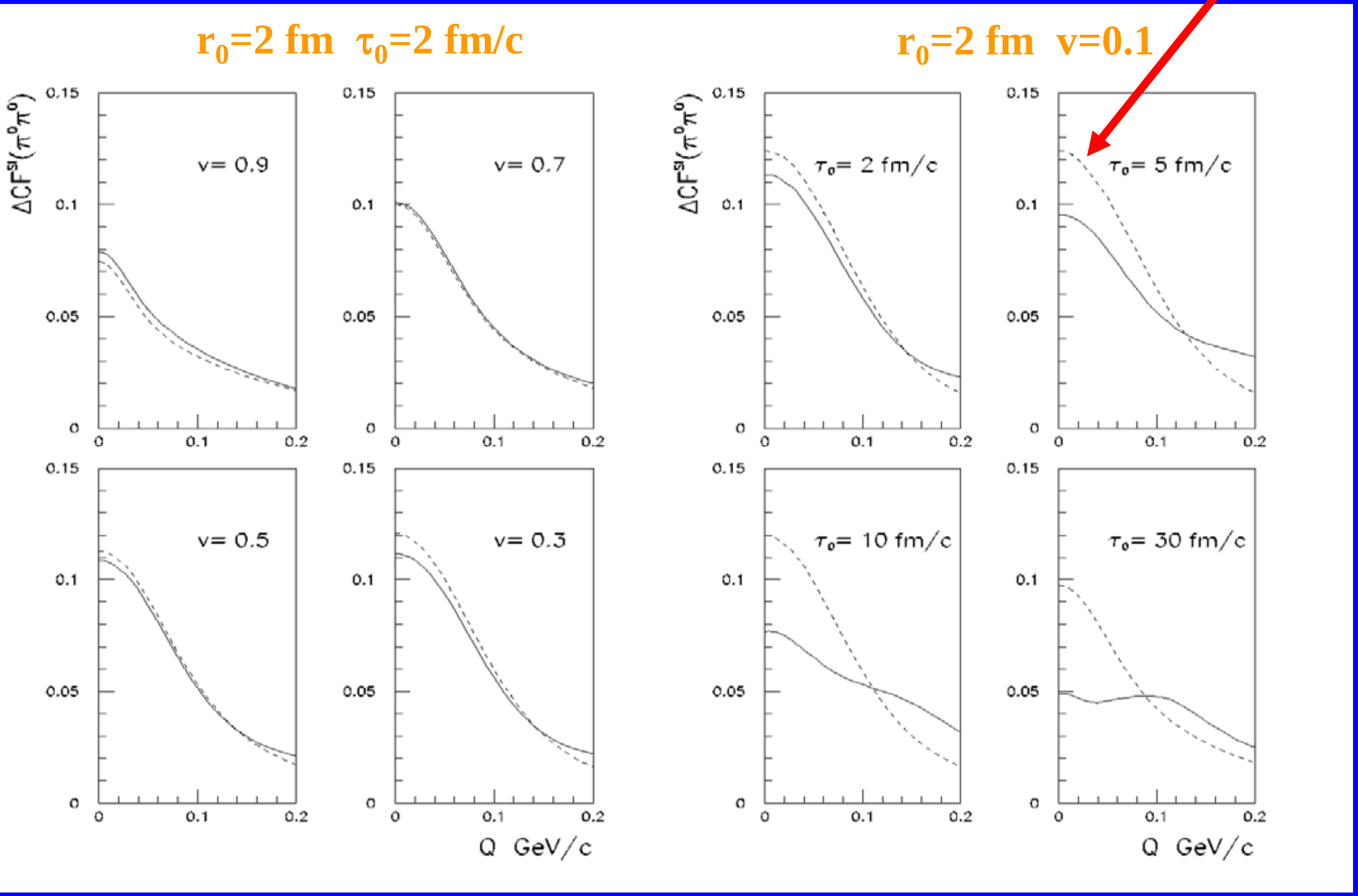
RL, Lyuboshitz SJNP 35 (82) 770;
RL nucl-th/0501065

$\Psi^{S(+)}_{p_1,p_2}(x_1,x_2) \rightarrow e^{iPX}\psi^{S}_{-\mathbf{k}^*}(\mathbf{r}^*)$

BS ampl. → WF

Applicability condition of equal-time approximation: $|t^*| << m_{1,2}r^{*2}$

$|k^*t^*| << m_{1,2}r^*$



↓

OK for heavy particles & small k*

→ OK within 5% even for pions if $\Delta\tau=\tau_0 \sim r_0$ or lower

"Fermi-like" CF formula

$$CF = \langle \, |\psi_{-k^*}(r^*)|^2 \, \rangle$$

Koonin'77: nonrelativistic & unpolarized protons

RL, Lyuboshitz'81: generalization to relativistic & polarized & nonidentical particles & estimated the effect of nonequal times

Assumptions:

- same as for KP formula in case of pure QS &
- equal time approximation in PRF

RL, Lyuboshitz'81 → eq. time conditions: $|t^*| << m_{1,2}r^{*2}$ $|k^*t^*| << m_{1,2}r^*$

OK (usually, to several % even for pions) fig.

- $t_{FSI} = d\delta/dE >> t_{prod}$

t_{FSI}(s-wave) $= \mu f_0/k^*$ → $|k^*| = ½|q^*| <<$ hundreds MeV/c ≈ typical momentum transfer in production

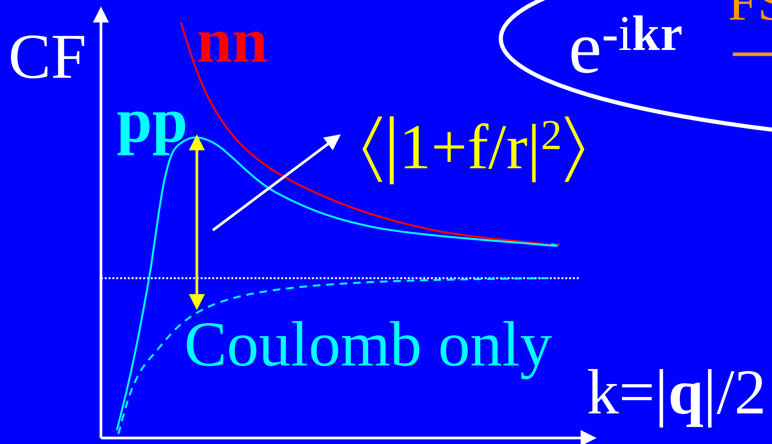
RL, Lyuboshitz ..'98

& account for coupled channels within the same isomultiplet only: $\pi^+\pi^- \leftrightarrow \pi^0\pi^0$, $\pi^- p \leftrightarrow \pi^0 n$, $K^+K^- \leftrightarrow K^0\bar{K}^0$, ...

Final State Interaction

Similar to Coulomb distortion of β-decay Fermi'34: $\langle|\psi_{-\mathbf{k}}(\mathbf{r})|^2\rangle$

Migdal, Watson, Sakharov, … Koonin, GKW, LL, ...



$$e^{-i\mathbf{kr}} \xrightarrow{\text{FSI}} \psi_{-\mathbf{k}}(\mathbf{r}) \xrightarrow[\text{s-wave}]{\text{strong FSI}} [\, e^{-i\mathbf{kr}} + f(k)e^{ikr}/r \,]$$

$f(k)$ → $f_c\sqrt{A_c}\cdot(G_0+iF_0)$

Coulomb: $e^{i\delta_c}\sqrt{A_c}$ — Point-like Coulomb factor

$$F = 1 + \frac{\mathbf{kr}+kr}{ka} + \ldots$$

a — Bohr radius

⇒ FSI is sensitive to source size r and scattering amplitude f

It **complicates CF analysis** but makes possible

→ **Femtoscopy with nonidentical particles** πK, πp, .. **& Coalescence** **deuterons**, ..

→ **Study "exotic" scattering** ππ, πK, KK, πΛ, pΛ, ΛΛ, ..

→ **Study relative space-time asymmetries** delays, flow

Using spherical wave in the outer region ($r>\varepsilon$) & inner region ($r<\varepsilon$) correction → **analytical dependence on s-wave scatt. amplitudes f_0 and source radius r_0** LL'81

⇒ FSI contribution to the CF of nonidentical particles, assuming Gaussian separation distribution $W(r)=\exp(-r^2/4r_0^2)/(2\sqrt{\pi}\, r_0)^3$

at $kr_0 \ll 1$:

$$\Delta CF^{FSI} = \tfrac{1}{2}|f_0/r_0|^2[1-d_0/(2r_0\sqrt{\pi})]+2f_0/(r_0\sqrt{\pi})$$

f_0 & d_0 are the s-wave scatt. length and eff. radius determining the scattering amplitude in the effective range approximation:

$$f_0(k) = \sin\delta_0 \exp(i\delta_0)/k \approx (1/f_0+\tfrac{1}{2}d_0k^2 - ik)^{-1}$$

f_0 and d_0 : characterizing the nuclear force

$u(r) = e^{i\delta} r \psi(r)$

$f_0 = -a$

$d_0 \approx r_0$

at $k \longrightarrow 0$

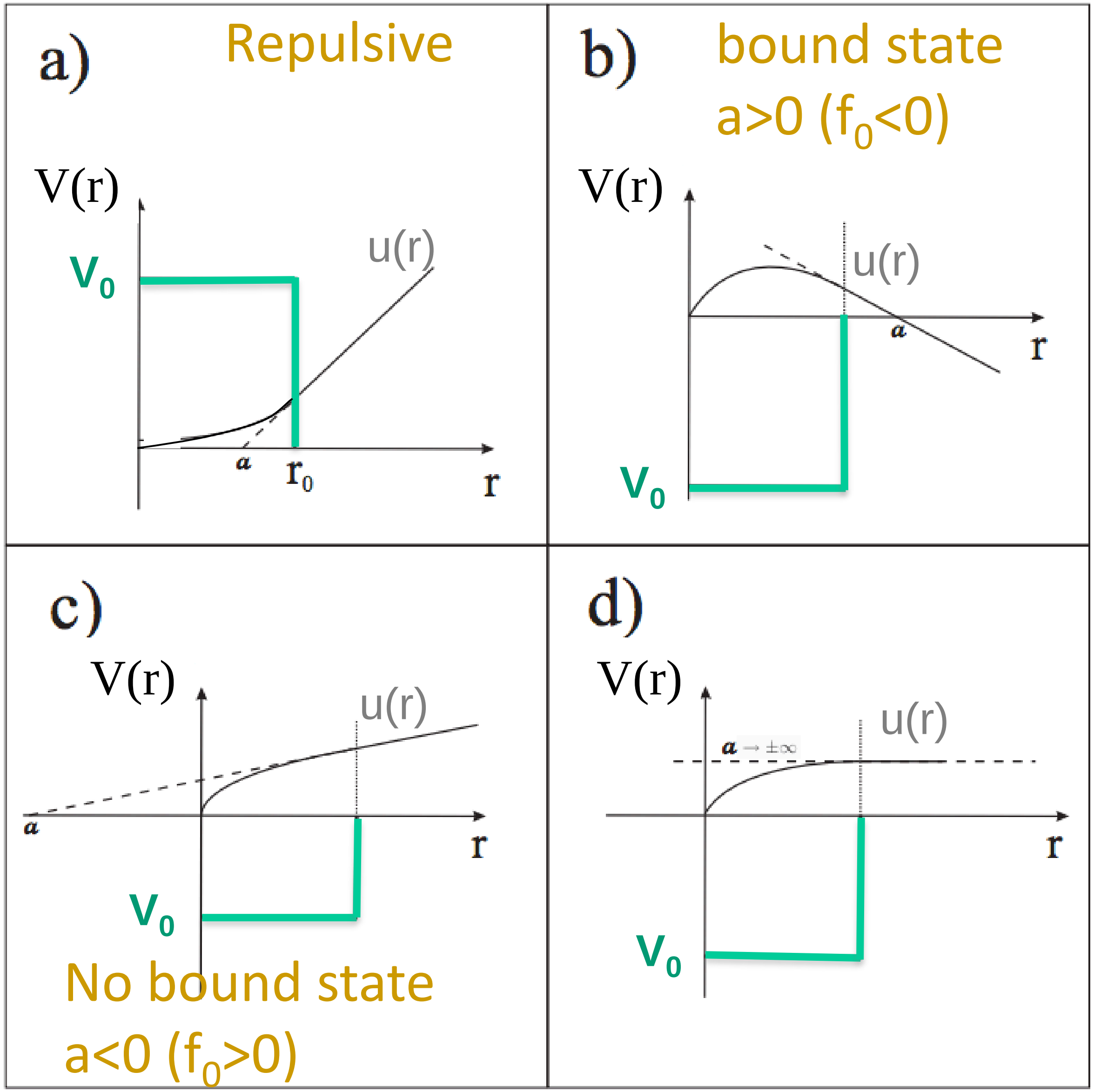
$r > r_0$

$u(r) \sim (r - a)$

Resonance:

$f_0 > 0$

$d_0 < 0$



f_0 and d_0 : How to measure them in scattering experiments

(not always possible with reasonable statistics)

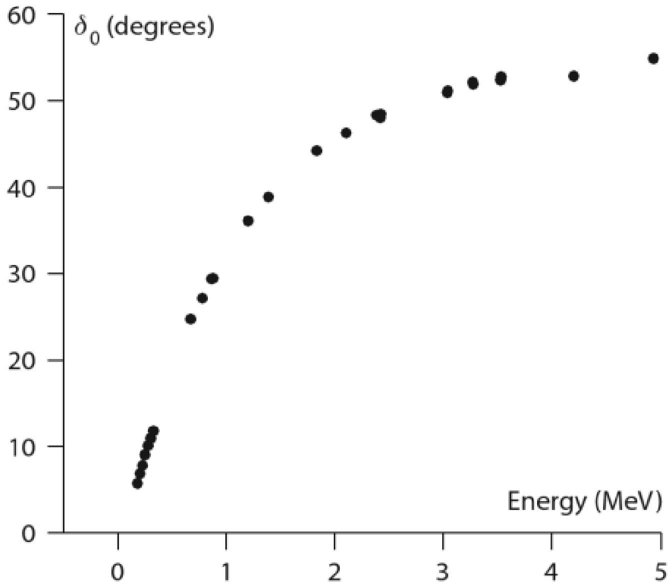


Figure 2.11 Phase shift variation as a function of the incident proton energy for proton-proton collision. The experimental points are from reference [JB50].

$$\frac{d\sigma}{d\Omega} = \left[\left(\frac{d\sigma}{d\Omega}\right)_c + \left(\frac{d\sigma}{d\Omega}\right)_n + \left(\frac{d\sigma}{d\Omega}\right)_{cn}\right]$$

Coulomb Nuclear Crossterm

$$\left(\frac{d\sigma}{d\Omega}\right)_c = \left(\frac{e^2}{2E_p}\right)^2 \left\{\frac{1}{\sin^4(\theta/2)} + \frac{1}{\cos^4(\theta/2)} - \frac{\cos\{\eta \ln[\tan^2(\theta/2)]\}}{\sin^2(\theta/2)\cos^2(\theta/2)}\right\}$$

$$\left(\frac{d\sigma}{d\Omega}\right)_n = \frac{\sin^2\delta_0}{k^2}$$

$$\left(\frac{d\sigma}{d\Omega}\right)_{cn} = -\frac{1}{2}\left(\frac{e^2}{E_p}\right)^2 \frac{\sin\delta_0}{\eta}\left\{\frac{\cos[\delta_0 + \eta\ln\sin^2(\theta/2)]}{\sin^2(\theta/2)} + \frac{\cos[\delta_0 + \eta\ln\cos^2(\theta/2)]}{\cos^2(\theta/2)}\right\}$$

"*Nuclear physics in a nutshell*",
Carlos A. Bertulani. Princeton U Press (2007).

$$k\cot(\delta_0) \approx \frac{1}{f_0} + \frac{1}{2} d_0 k^2$$

f_0 and d_0 can be extracted by studying the phase shift vs. energy.

Correlation analysis:

a possibility to measure f_0 & d_0 for any abundantly produced particle pairs

Correlation Function (CF):

$$C(p_1, p_2) = \frac{P(p_1, p_2)}{P(p_1)P(p_2)}$$

In practice,

$$C(k^*)_{measured} = \frac{\text{real pairs from same events}}{\text{pairs from mixed events}}$$

Purity correction (misidentification + ? weak decays) :

$$C(k^*) = \frac{C(k^*)_{measured} - 1}{\text{PairPurity}(k^*)} + 1$$

Correlation femtoscopy with nonid. particles

pΛ CFs at AGS & SPS & STAR

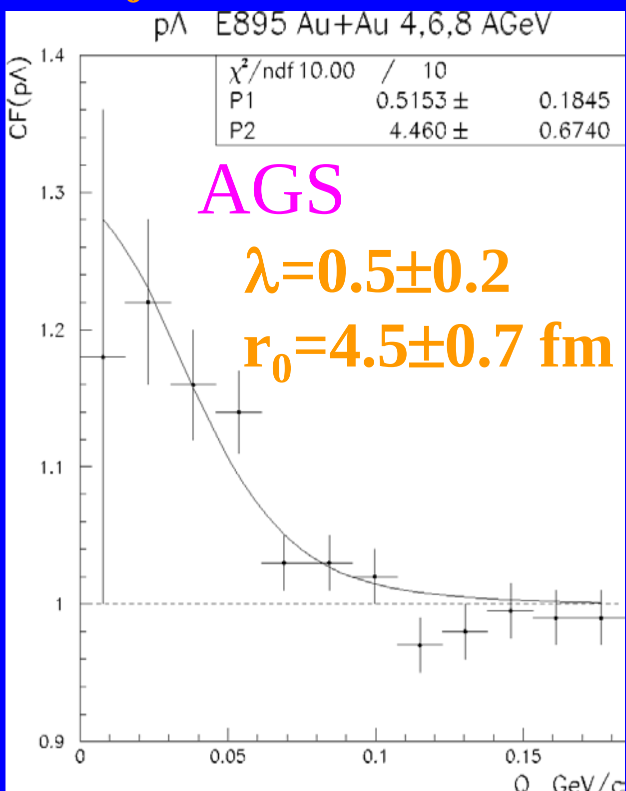
Goal: No Coulomb suppression as in pp CF &
Wang-Pratt'99 Stronger sensitivity to r_0

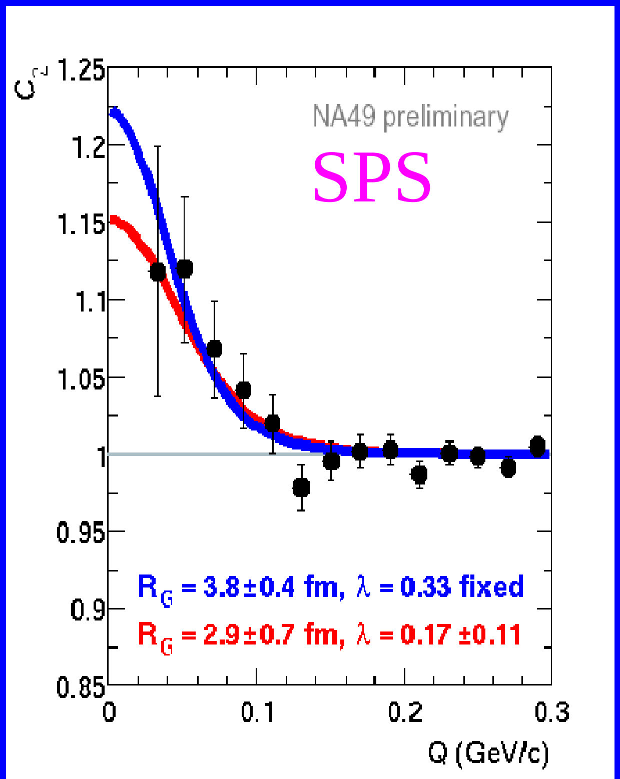
	singlet	triplet
Scattering lengths, fm:	2.31	1.78
Effective radii, fm:	3.04	3.22

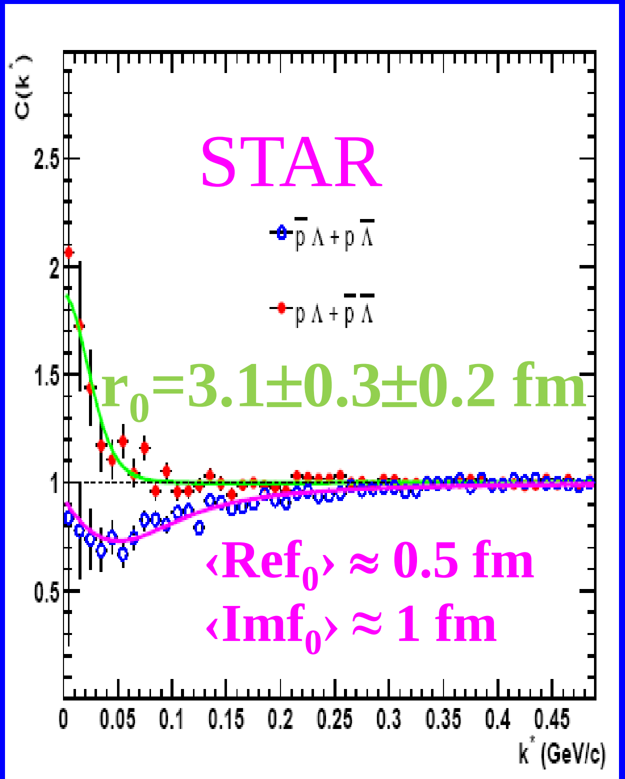
Fit using RL-Lyuboshitz'82 with

λ consistent with estimated impurity

r_0~ 3-4 fm consistent with the radius from pp CF & m_t scaling







Pair purity problem for pΛ CF @ STAR

Particle	Identification	Fraction Primary
p	$76 \pm 7\%$	$52 \pm 4\%$
$\overline{p}$	$74 \pm 7\%$	$48 \pm 4\%$
Λ	$86 \pm 6\%$	$45 \pm 4\%$
$\overline{\Lambda}$	$86 \pm 6\%$	$45 \pm 4\%$

⇒ **PairPurity ~ 15%**

Assuming no correlation for misidentified particles and particles from weak decays

→ $$C^{corr}_{measured}(k^*) = \frac{C_{measured}(k^*) - 1}{\mathrm{PairPurity}} + 1$$

$$C(k^*) = 1 + \sum_S \rho_S \left[\frac{1}{2} \left| \frac{f^S(k^*)}{r_0} \right|^2 \left(1 - \frac{d_0^S}{2\sqrt{\pi} r_0} \right) + \frac{2\Re f^S(k^*)}{\sqrt{\pi} r_0} F_1(Qr_0) - \frac{\Im f^S(k^*)}{r_0} F_2(Qr_0) \right],$$

where $F_1(z) = \int_0^z dx e^{x^2 - z^2}/z$ and $F_2(z) = (1 - e^{-z^2})/z$.

$$f^S(k^*) = \left(\frac{1}{f_0^S} + \frac{1}{2} d_0^S k^{*2} - ik^* \right)^{-1}$$

← Fit using RL-Lyuboshitz'82 (for np)

Pairs	Fractions (%)
p_{prim}-Λ_{prim}	15
p_{Λ}-Λ_{prim}	10
p_{Σ^+}-Λ_{prim}	3
p_{prim}-Λ_{Σ^0}	11
p_{Λ}-Λ_{Σ^0}	7
p_{Σ^+}-Λ_{Σ^0}	2
p_{prim}-Λ_{Ξ}	9
p_{Λ}-Λ_{Ξ}	5
p_{Σ^+}-Λ_{Ξ}	2

← but, there can be residual correlations for particles from weak decays requiring knowledge of ΛΛ, pΣ, ΛΣ, ΣΣ, pΞ, ΛΞ, ΣΞ correlations

Correlation study of strong interaction

$\pi^+\pi^-$ & $\Lambda\Lambda$ & $\bar{p}\Lambda$ & $\bar{p}\bar{p}$ s-wave scattering parameters from NA49 and STAR

Fits using RL-Lyuboshitz'82

$\bar{p}\Lambda$: STAR data accounting for residual correlations
- Kisiel et al, PRC 89 (2014) : assuming a universal $\mathrm{Im}f_0$
- Shapoval et al, PRC 92 (2015): Gauss. parametr. of res. CF

$\mathrm{Re}f_0 \approx 0.5$ fm, $\mathrm{Im}f_0 \approx 1$ fm, $r_0 \approx 3$ fm

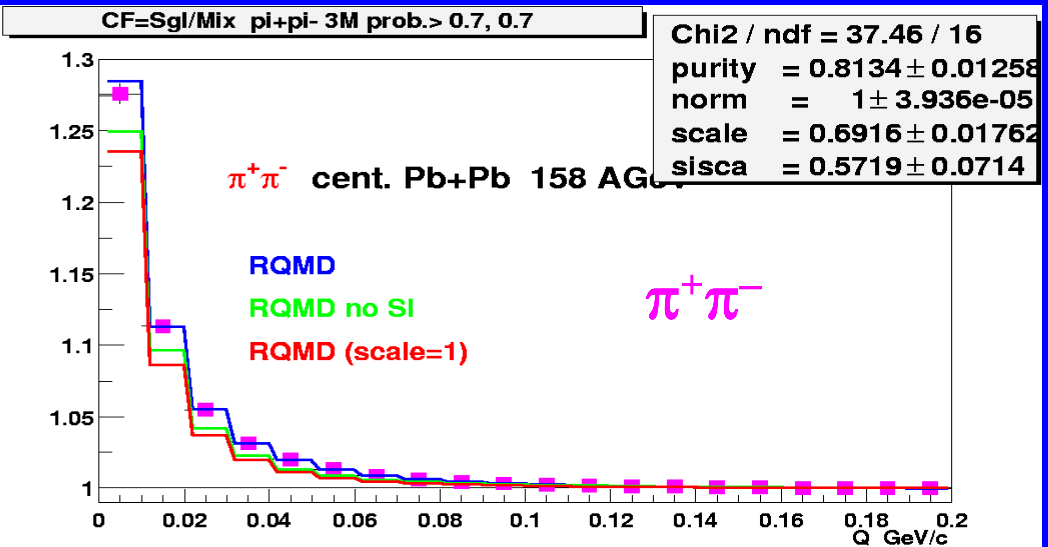
$\Lambda\Lambda$: NA49: $|f_0(\Lambda\Lambda)| << f_0(NN) \sim 20$ fm
STAR, PRL 114 (2015): $f_0(\Lambda\Lambda) \approx -1$ fm, $d_0(\Lambda\Lambda) \approx 8$ fm

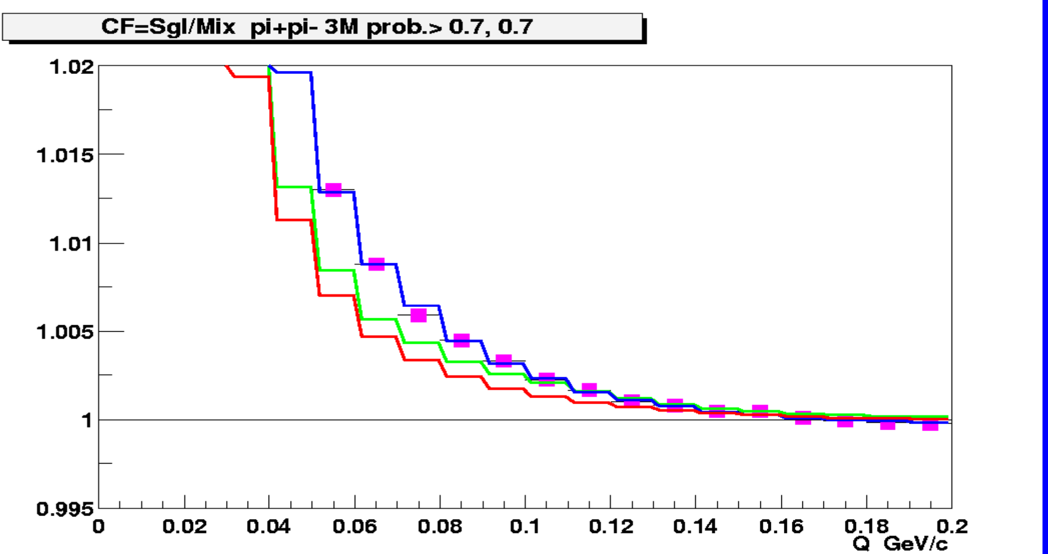
$\pi^+\pi^-$: NA49 vs RQMD with SI scale: $f_0 \rightarrow$ sisca f_0 (=0.232fm)
sisca = 0.6±0.1 compare with
~0.8 from SχPT & BNL data E765 $K \rightarrow e\nu\pi\pi$
Here a (2 s.d.) suppression can be due to eq. time approx.

$\bar{p}\bar{p}$: STAR, Nature (2015): f_0 and d_0 coincide with PDG table pp-values

Correlation study of particle interaction

CF=Norm [Purity RQMD(r* → Scale•r*)+1-Purity]





$\pi^+\pi^-$ scattering length f_0 from NA49 CF

Fit CF($\pi^+\pi^-$) by RQMD with SI scale:

$f_0 \rightarrow$ sisca f_0^{input}

f_0^{input} = 0.232 fm

sisca = 0.6±0.1

to be 0.8

from SχPT

& BNL E765

$K \rightarrow e\nu\pi\pi$

Correlation study of strong interaction

$\Lambda\Lambda$ scattering parameters f_0 & d_0 from STAR correlation data

Fit using RL-Lyuboshitz (81):

$$CF=1+\lambda[\Delta CF^{FSI}+\Sigma_S\rho_S(-1)^S\exp(-r_0^2Q^2)]$$

$$+\,a_{res}\exp(-r_{res}^2Q^2)$$

$\rho_0 = \frac{1}{4}(1-P^2)$ $\rho_1 = \frac{1}{4}(3+P^2)$ $P=\text{Polar.}=0$

$$\Delta CF^{FSI} = 2\rho_0[\tfrac{1}{2}|f^0(k)/r_0|^2(1-d_0^0/(2r_0\sqrt{\pi}))$$

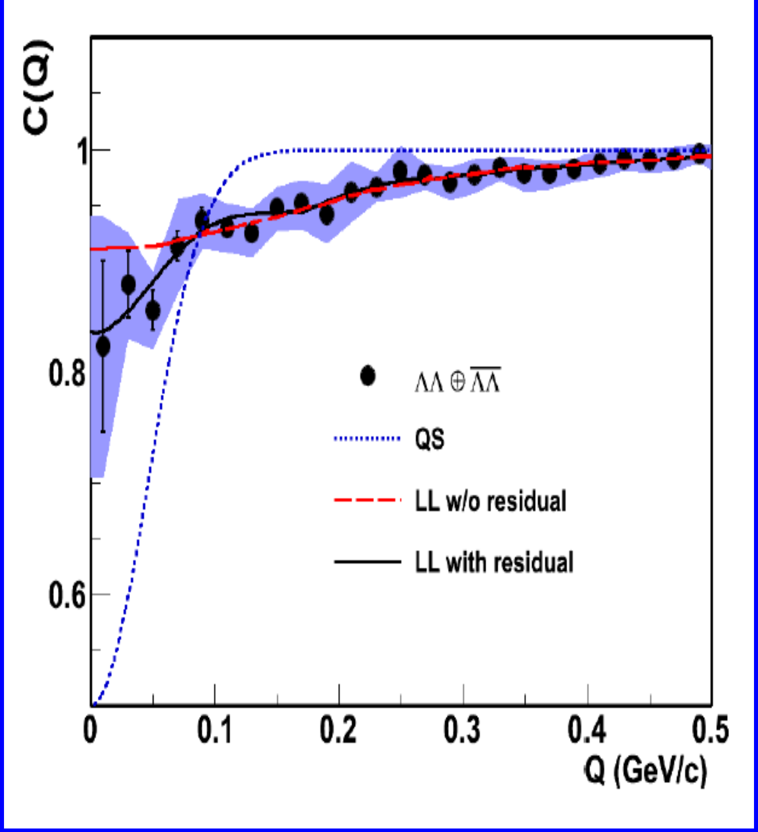
$$+2\text{Re}(f^0(k)/(r_0\sqrt{\pi}))F_1(r_0Q)$$

$$-\text{Im}(f^0(k)/r_0)F_2(r_0Q)]$$

$f^S(k)=(1/f_0^S+\frac{1}{2}d_0^S k^2 - ik)^{-1}$ $k=Q/2$

$F_1(z)=\int_0^z dx\,\exp(x^2-z^2)/z$

$F_2(z)=[1-\exp(-z^2)]/z$



$\lambda \approx 0.18$, $r_0 \approx 3$ fm,

$a_{res} \approx -0.04$, $r_{res} \approx 0.4$ fm

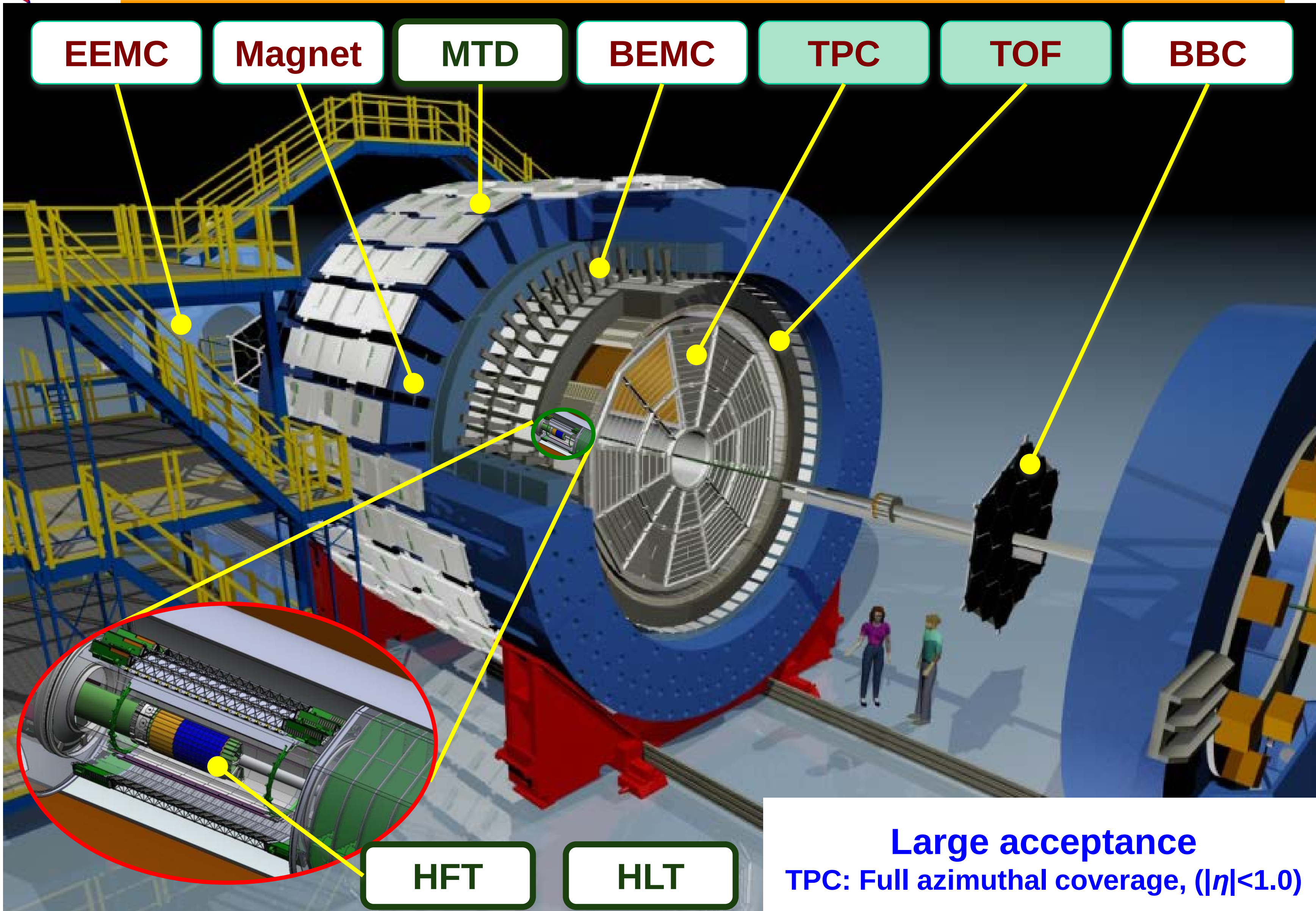
$f_0 \approx -1$ fm, $d_0 \approx 8$ fm $\Rightarrow$

s-wave resonance: excluded

Bound state: possible

Correlation study of strong pp & $\bar{p}\bar{p}$ interaction at STAR

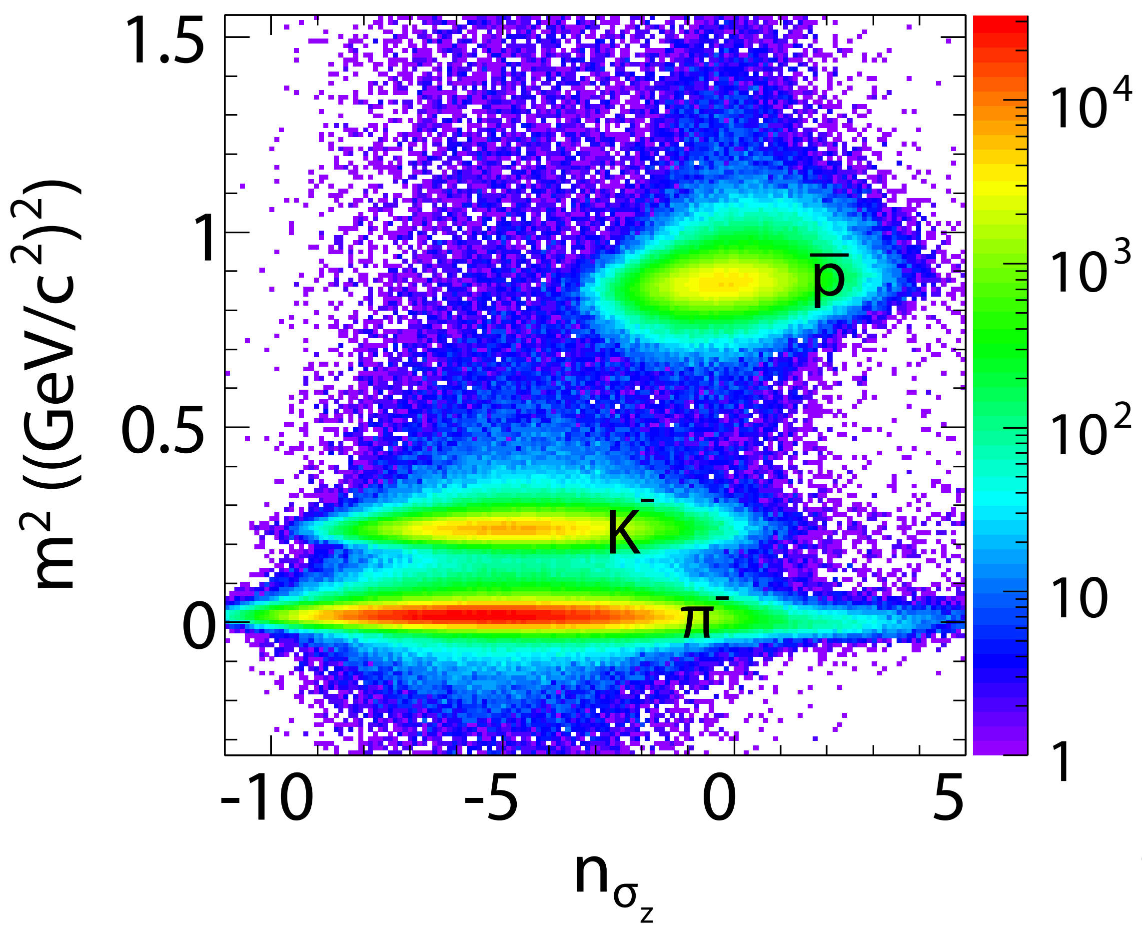
STAR detector complex



Large acceptance

TPC: Full azimuthal coverage, ($|\eta|<1.0$)

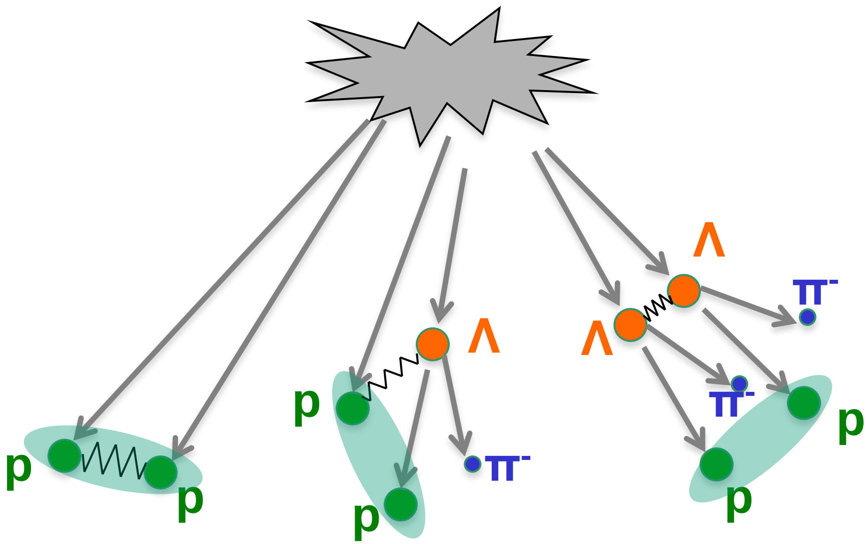
Particle Identification



$$z = \ln \frac{\langle dE/dx \rangle}{\langle dE/dx \rangle_{expected}}$$

PID by Time Projection Chamber (TPC) and Time of Flight detector (TOF). Purity for anti-protons is over 99%.

Residual pp correlation



The observed (anti)protons can come from weak decays of already correlated primary particles, hence introducing residual correlations which contaminate the CF
(generally cannot be treated as a constant impurity).

Residual correlation

Taking dominant contributions due to residual correlation, the measured correlation function can be expressed as :

$$C_{measured}(k^*) = 1 + x_{pp}\left[C_{pp}(k^*;R_{pp}) - 1\right] + x_{p\Lambda}\left[\tilde{C}_{p\Lambda}(k^*;R_{p\Lambda}) - 1\right] + x_{\Lambda\Lambda}\left[\tilde{C}_{\Lambda\Lambda}(k^*;R_{\Lambda\Lambda}) - 1\right]$$

where

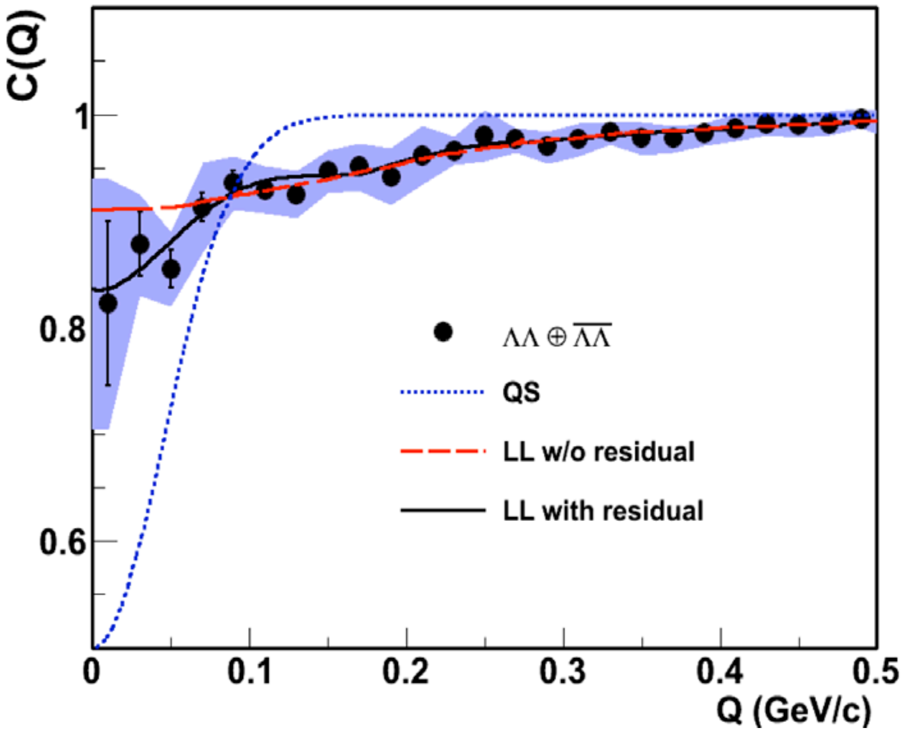
$$\tilde{C}_{p\Lambda}(k^*_{pp}) = \int C_{p\Lambda}(k^*_{p\Lambda})T(k^*_{p\Lambda}, k^*_{pp})dk^*_{p\Lambda}$$

$$\tilde{C}_{\Lambda\Lambda}(k^*_{pp}) = \int C_{\Lambda\Lambda}(k^*_{\Lambda\Lambda})T(k^*_{\Lambda\Lambda}, k^*_{pp})dk^*_{\Lambda\Lambda}$$

	DCA	x_{pp}	$x_{p\Lambda}$	$x_{\Lambda\Lambda}$
proton-proton	$2cm$	0.45	0.375	0.077
proton-proton	$1cm$	0.51	0.335	0.055
pbar-pbar	$2cm$	0.42	0.385	0.092
pbar-pbar	$1cm$	0.485	0.35	0.063

- $C_{pp}(k^*)$ and $C_{p\Lambda}(k^*_{p\Lambda})$ are calculated by the Lednicky and Lyuboshitz model.
- $C_{\Lambda\Lambda}(k^*_{\Lambda\Lambda})$ is from STAR publication (PRL 114 22301 (2015)).
- Regard $R_{p\Lambda}$ and $R_{\Lambda\Lambda}$ are equal to R_{pp}.
- T is the corresponding tranform matirces, generated by THERMINATOR2, to transform $k^*_{p\Lambda}$ to k^*_{pp}, as well as $k^*_{\Lambda\Lambda}$ to k^*_{pp}.

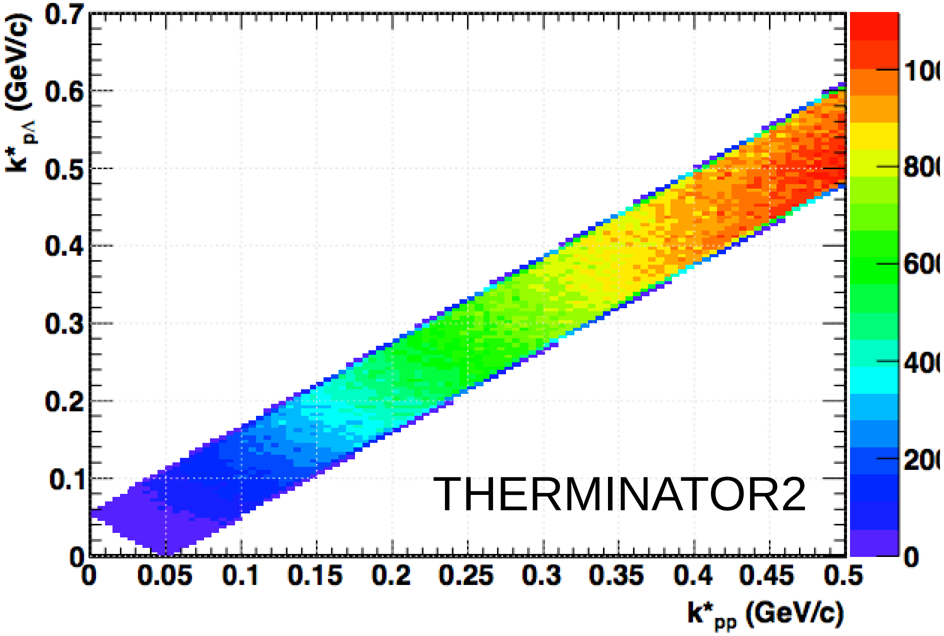
Residual from Λ-Λ correlation

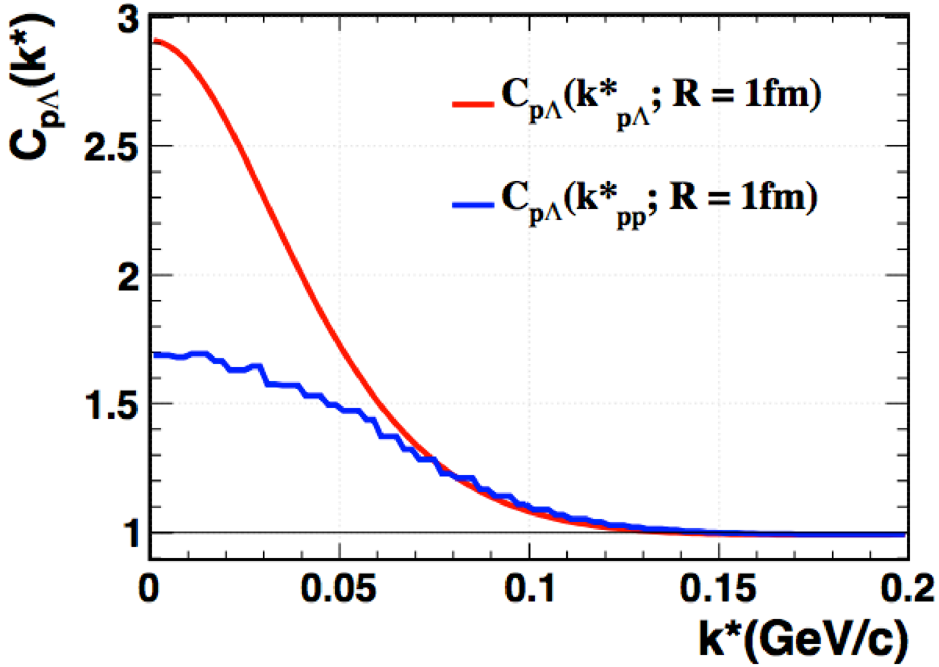


STAR Collaboration (PRL 114 22301 (2015)).

- $C_{\Lambda\Lambda}(k^*_{\Lambda\Lambda})$ CF is taken from experimental input

Transformation Matrix





$$\tilde{C}_{p\Lambda}(k^*_{pp}) = \int C_{p\Lambda}(k^*_{p\Lambda})T(k^*_{p\Lambda}, k^*_{pp})dk^*_{p\Lambda}$$

$$\tilde{C}_{\Lambda\Lambda}(k^*_{pp}) = \int C_{\Lambda\Lambda}(k^*_{\Lambda\Lambda})T(k^*_{\Lambda\Lambda}, k^*_{pp})dk^*_{\Lambda\Lambda}$$

Connecting f_0 & d_0 to CF

The theoretical correlation function can be obtained with

$$C(k^*) = \frac{\sum_{pairs} \delta(k^*_{pairs} - k^*) w(k^*, r^*)}{\sum_{pairs} \delta(k^*_{pairs} - k^*)}$$

where $w(k^*, r^*) = \left| \psi^{S(+)}_{-k^*}(r^*) + (-1)^S \psi^{S(+)}_{k^*}(r^*) \right|^2 / 2$ and

$$\psi^{S(+)}_{-k^*}(r^*) = e^{i\delta_c} \sqrt{A_c(\eta)} \left[e^{-ik^* r^*} F(-i\eta, 1, i\xi) + f_c(k^*) \frac{\tilde{G}(\rho, \eta)}{r^*} \right]$$

$$f_c(k^*) = \left[\frac{1}{f_0} + \frac{1}{2} d_0 k^{*2} - \frac{2}{a_c} h(\eta) - ik^* A_c(\eta) \right]^{-1}$$ is the s-wave scattering amplitude

renormalized by Coulomb interaction.

$\eta = (k^* a_c)^{-1}$, $a_c = 57.5$ fm

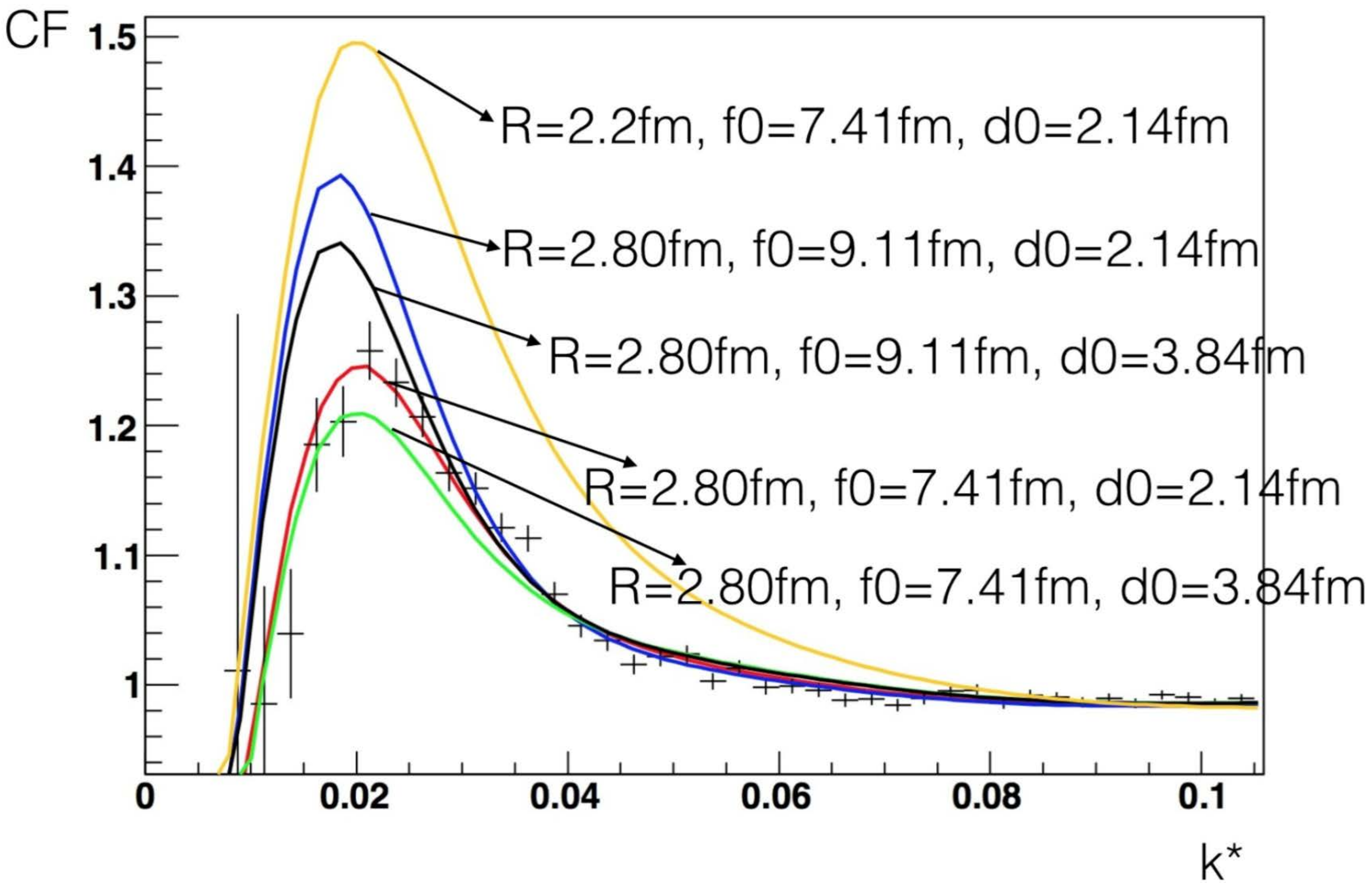
$\rho = k^* r^*$, $\xi = k^* r^* + \rho$

$A_c(\eta) = 2\pi\eta \left[\exp(2\pi\eta) - 1 \right]^{-1}$

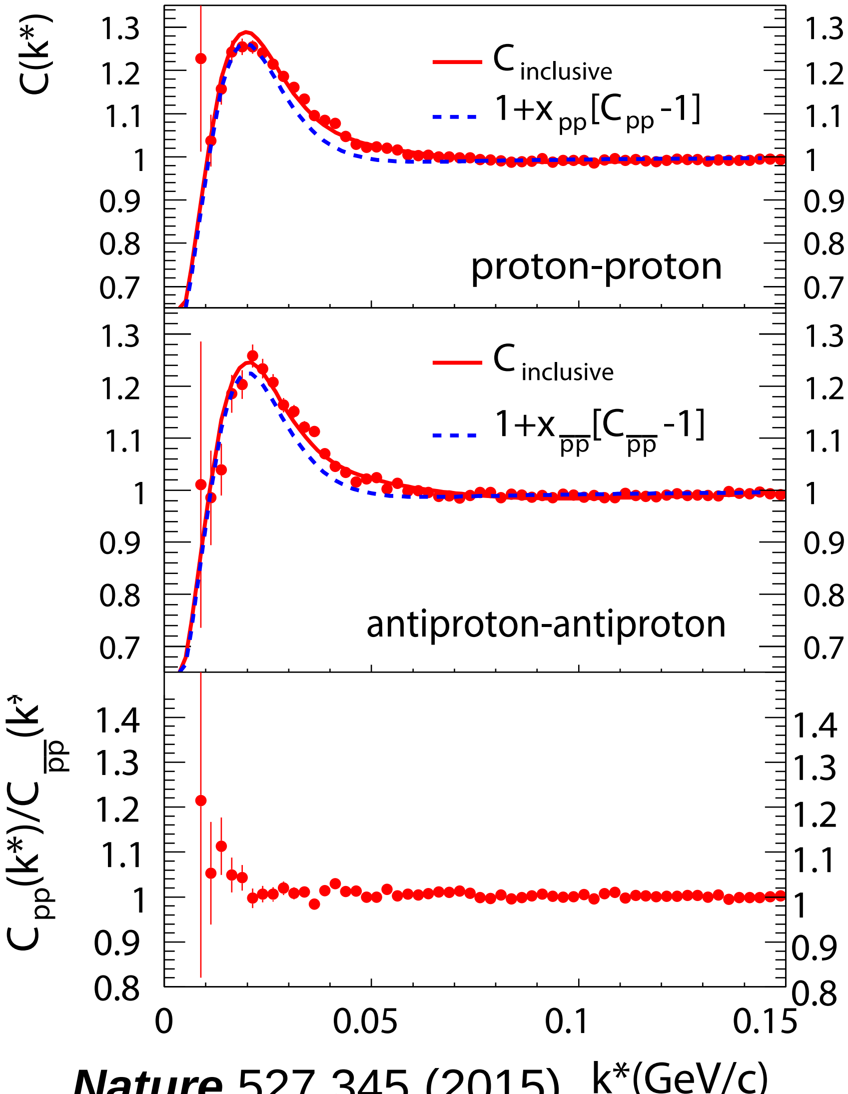
F is the confluent hypergeometric function

$\tilde{G}(\rho, \eta) = \sqrt{A_c(\eta)} \left[G_0(\rho, \eta) + iF_0(\rho, \eta) \right]$ is a combination of the regular (F_0) and singular (G_0) s-wave Coulomb functions. Proton pairs are from THERMINATOR2 when deriving theoretical C(K*)

Connecting f_0 & d_0 to CF



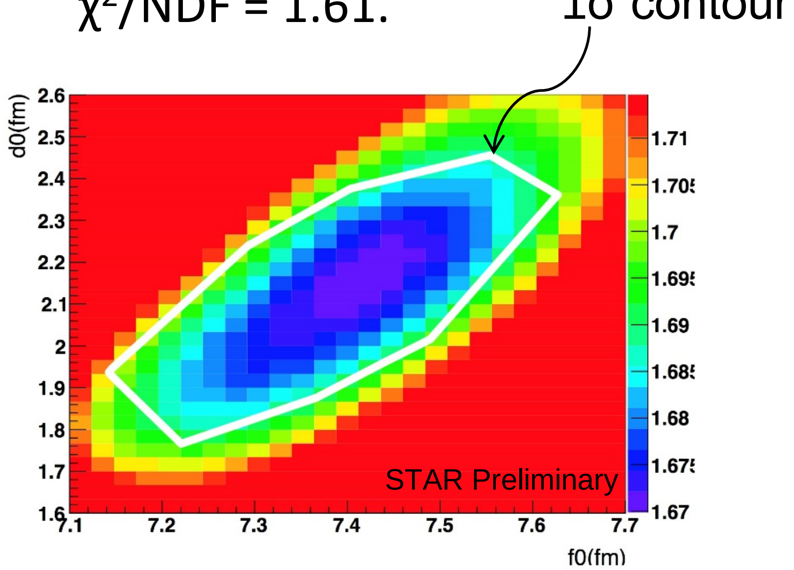
Correlation functions



Nature 527 345 (2015)

- For proton-proton CF
  $R = 2.75 \pm 0.01$ fm;
  $\chi^2/NDF = 1.66$.

- For antiproton-antiproton CF
  $R = 2.80 \pm 0.02$ fm;
  $f_0 = 7.41 \pm 0.19$ fm;
  $d_0 = 2.14 \pm 0.27$ fm;
  $\chi^2/NDF = 1.61$.



Main systematics

The decomposition of systematics of this analysis:

	Δf_0 (± fm)	Δd_0 (± fm)	$\Delta R_{\bar{p}\bar{p}}$ (± fm)	ΔR_{pp} (± fm)
experimental cuts	0.14	0.33	0.01	0.03
uncertainty of p-Λ CF	0.17	0.19	0.03	0.01
uncertainty of Λ-Λ CF	0.36	1.34	0.03	0.03
THERMINATOR2 model	0.07	0.09	< 0.01	< 0.01

Final systematics is given by (max-min)/√12

Other systematics that are not considered in this analysis :

- Non-Coulomb electromagnetic contribution due to magnetic interactions
- Vacuum polarization
- Finite proton size

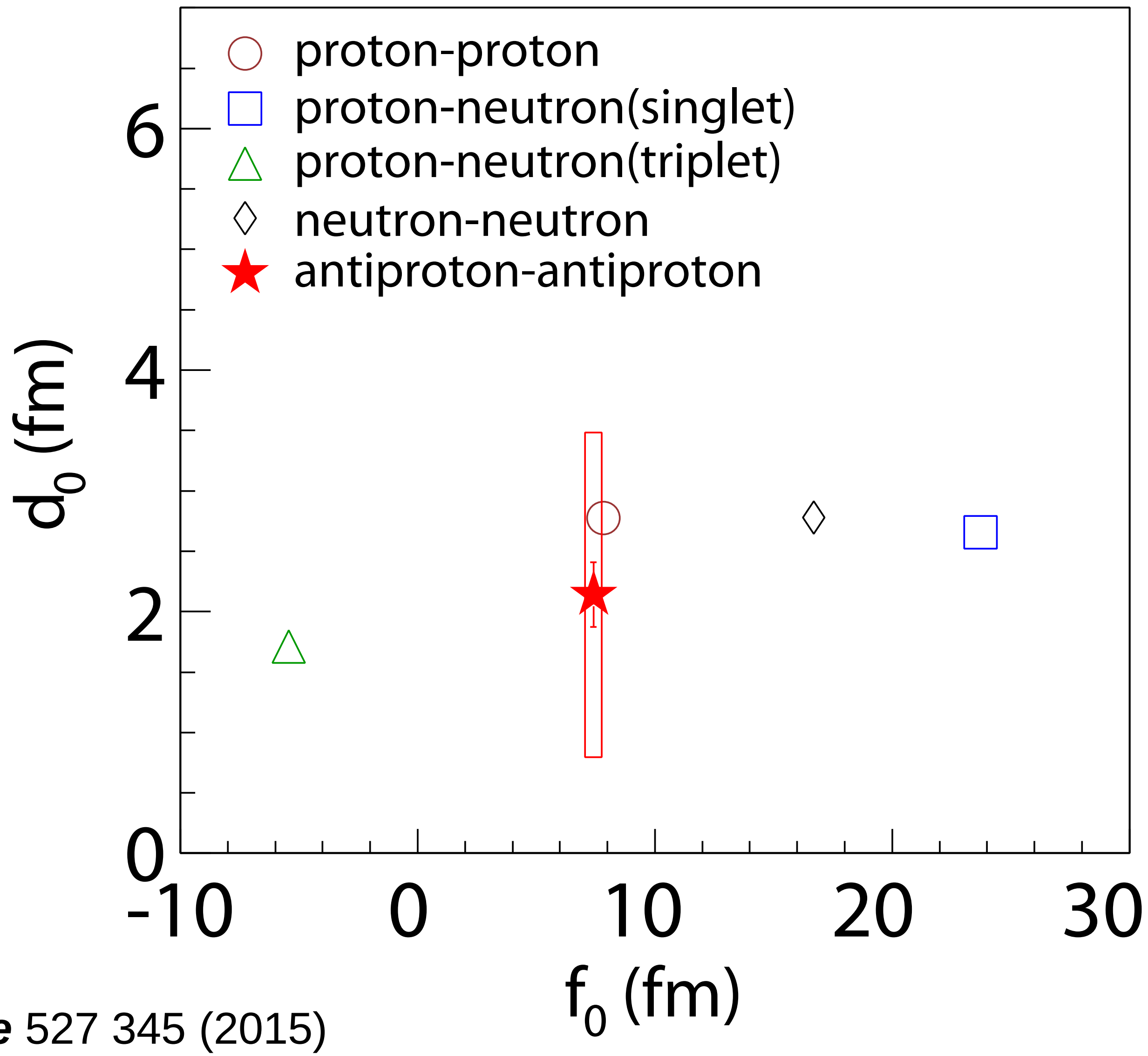
These effects change the f_0 and d_0 at the level of a few percent in total.

L. Mathelitsch and B. J. VerWest, *Phys. Rev. C* 29, 739-745 (1984).

L. Heller *Rev. of Mod. Phys.* 39, 584-590 (1967).

J. R. Bergervoet, P.C. van Campen W.A. van der Sanden, and J.J. de Swart, *Phys. Rev. C* 38, 15-50 (1988)

f_0 and d_0



Nature 527 345 (2015)

"This paper announces an important discovery! … offers important original contribution to the forces in antimatter!" – *Nature* Referee A

"… significance of the results can be considered high since this is really the first and only result available on the interaction between the antiprotons ever." – *Nature* Referee B

"… are of fundamental interest for the whole nuclear physics community and possible even beyond for atomic physics applications or condensed matter physicists. … I think that this paper is most likely one of the five most significant papers published in the discipline this year" – *Nature* Referee C

Summary

- Assumptions behind femtoscopy theory in HIC OK at $k \to 0$ (in particular, correlation measurement recovers table values of pp scattering parameters).

- Wealth of data on correlations of various particles ($\pi^{\pm}$,$K^{\pm 0}$,$p^{\pm}$,Λ,Ξ) is available & gives unique space-time info on production characteristics thanks to the effects of QS and FSI.

- Info on two-particle s-wave strong interaction of abundantly produced particles:
  $\pi\pi$ & $\Lambda\Lambda$ & $\bar{p}\Lambda$ & $\bar{p}\bar{p}$ scattering amplitudes
  from HIC at SPS and RHIC
  (on a way to solving the problem of residual correlations).
  A good perspective: high statistics RHIC & LHC data.

Spare Slides

Phase space density from CFs and spectra

$$\bar{f}(p_t) \equiv \frac{\int d^3r\ f(p,r)\cdot f(p,r)}{\int d^3r\ f(p,r)}$$

$$\sim \frac{1}{2\pi m_\pi}\frac{dN}{p_t dp_t dy}\int d^3Q_{inv}C(p_t,Q_{inv})$$

$$f_{max}(p_t,r) \approx 2\sqrt{2}\bar{f}(p_t)$$

Bertsch'94

Lisa ..'05

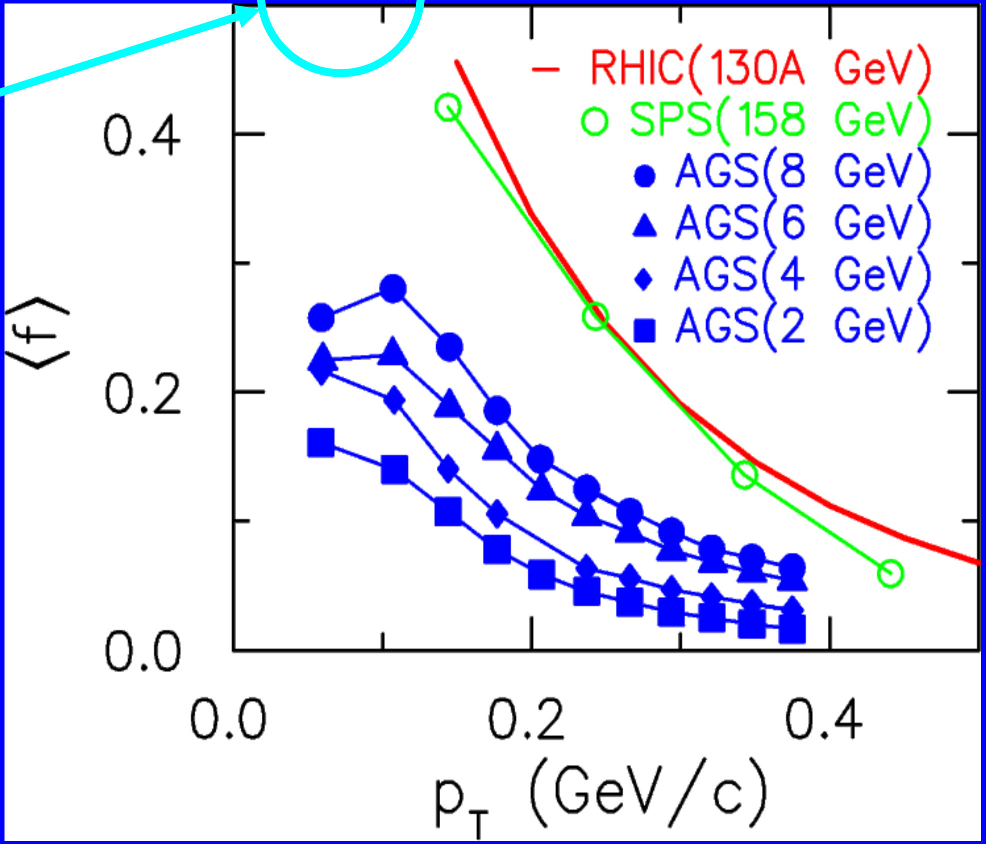
$\langle f \rangle$ rises up to SPS

May be high phase space density at low p_t ?

↓

? Pion condensate or laser

? Multiboson effects on CFs spectra & multiplicities



Gyulassy, Kaufmann, Wilson 1979

Plane wave $\xrightarrow{\text{FSI}}$ Bethe-Salpeter amplitude

$$\exp(-ip_1x_1-ip_2x_2) \rightarrow \psi_{p1p2}(x_1,x_2)$$

In pair CMS, only relative quantities are relevant: $q=\{0,2\mathbf{k}\}$, $\Delta x=\{t,\mathbf{r}\}$

$$\exp(ikr) \rightarrow \psi_q(t,r)$$

at $t = 0$, the reduced B-S ampl. coincides with the usual WF:

$$\psi_q(t=0,r) = [\psi_{-k}(r)]^*$$

Note: in beta-decay $A \rightarrow A' + e + \nu$
$t(A')-t(e) = 0$ in A rest frame $\approx$ t in A'e-pair rest frame

Lednicky, Lyuboshitz 1981

- Eq. time approximation $t=0$ is valid on condition $|t| \ll m_{1,2}r^2$
  Usually OK to several % even for pions
- Smoothness approx. applied also to non-id. particles

Note

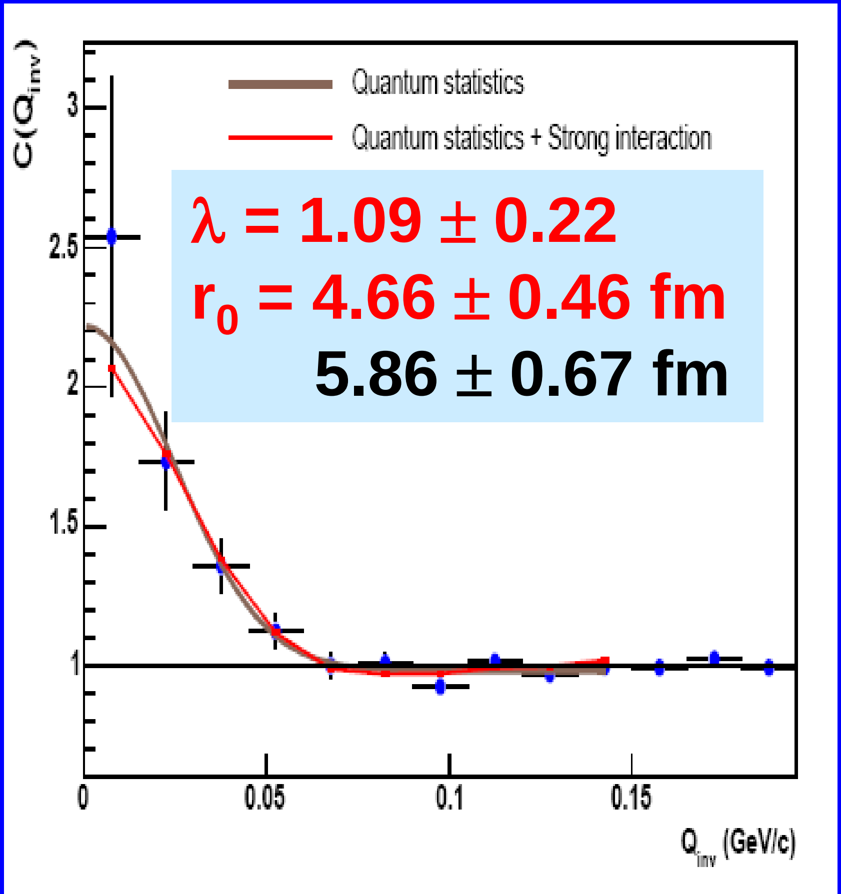
- Formally (FSI) correlations in beta decay and multiparticle production are determined by the same (Fermi) function $\langle|\psi_{-\mathbf{k}}(\mathbf{x})|^2\rangle$

- But it appears for different reasons in
  beta decay: a weak $\mathbf{r}$-dependence of $\psi_{-\mathbf{k}}(\mathbf{r})$ within the nucleus volume + point like + equal time emission and in
  multiparticle production in usual events of HIC: a small space-time extent of the emitters compared to their separation + sufficiently small phase space density + a small effect of nonequal emission times in usual conditions

FSI effect on CF of neutral kaons

Lyuboshitz-Podgoretsky'79:
K_sK_s from $K\bar{K}$ also show BE enhancement

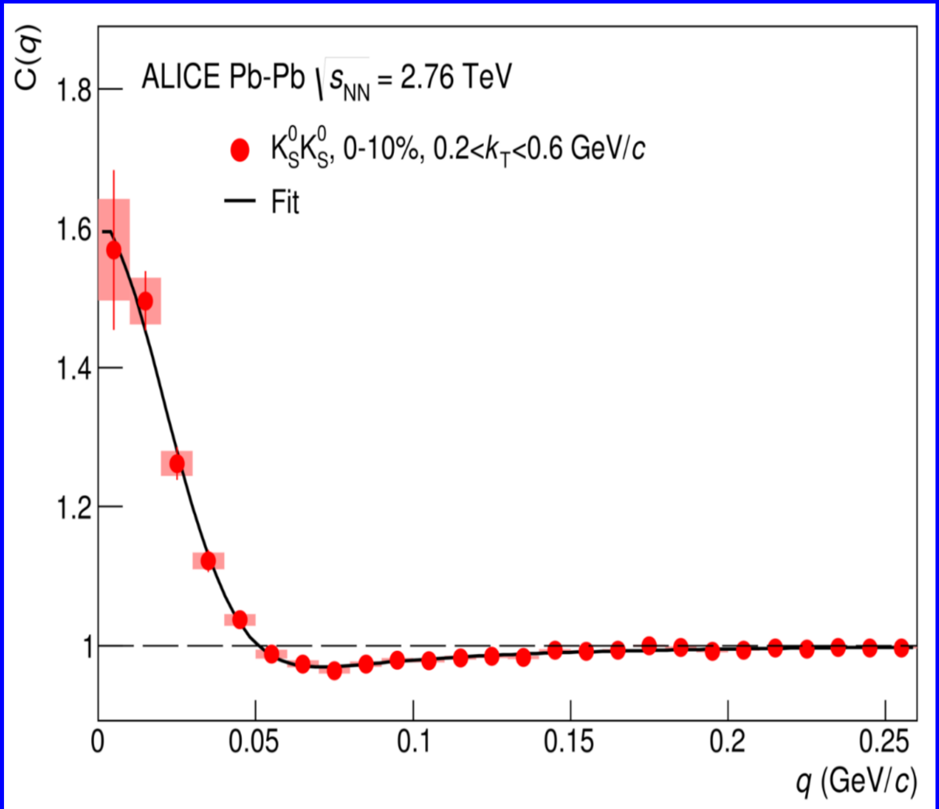
STAR data on CF(K_sK_s)
arXiv:1206.2056



Goal: no Coulomb. But R (λ) may go up to 40 (100)% if neglecting FSI in $K\bar{K}$ (~50% K_sK_s)↔ $f_0(980)$ & $a_0(980)$

RL-Lyuboshitz'81

ALICE data on CF(K_sK_s)
arXiv.org:1506.07884



Even stronger effect of KK-bar FSI on K_sK_s correlations in pp-collisions at LHC

ALICE: PLB 717 (2012) 151

e.g. for $k_t < 0.85$ GeV/c, N_{ch}=1-11 the neglect of FSI increases λ by ~100% and R_{inv} by ~40%

$\lambda = 0.64 \pm 0.07 \rightarrow 1.36 \pm 0.15 > 1$!

$R_{inv} = 0.96 \pm 0.04 \rightarrow 1.35 \pm 0.07$ fm

NA49 central Pb+Pb 158 AGeV vs RQMD: FSI theory OK

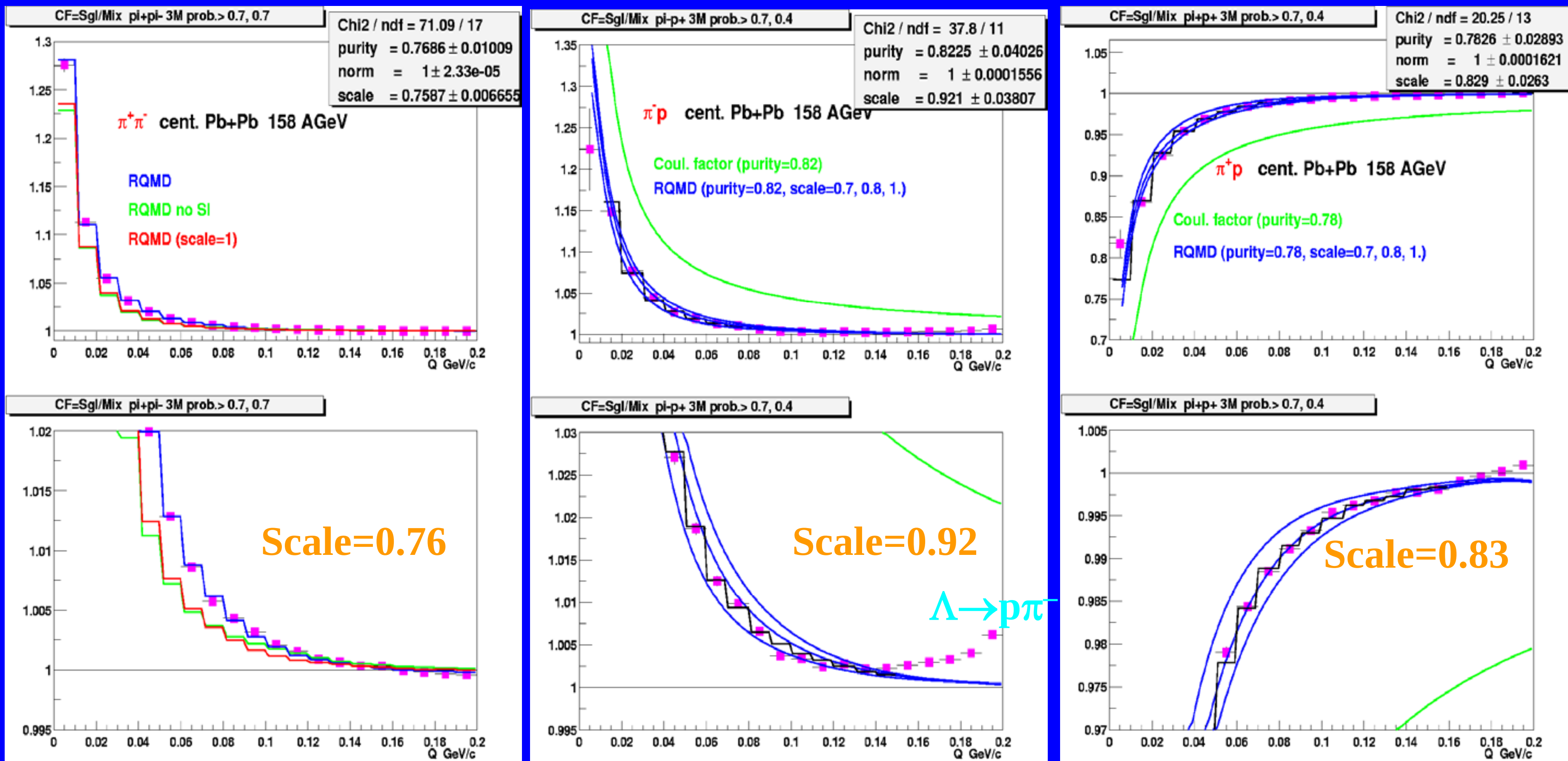
Long tails in RQMD: $\langle r^* \rangle$ = **21** fm for r^* < **50** fm

29 fm for r^* < **500** fm

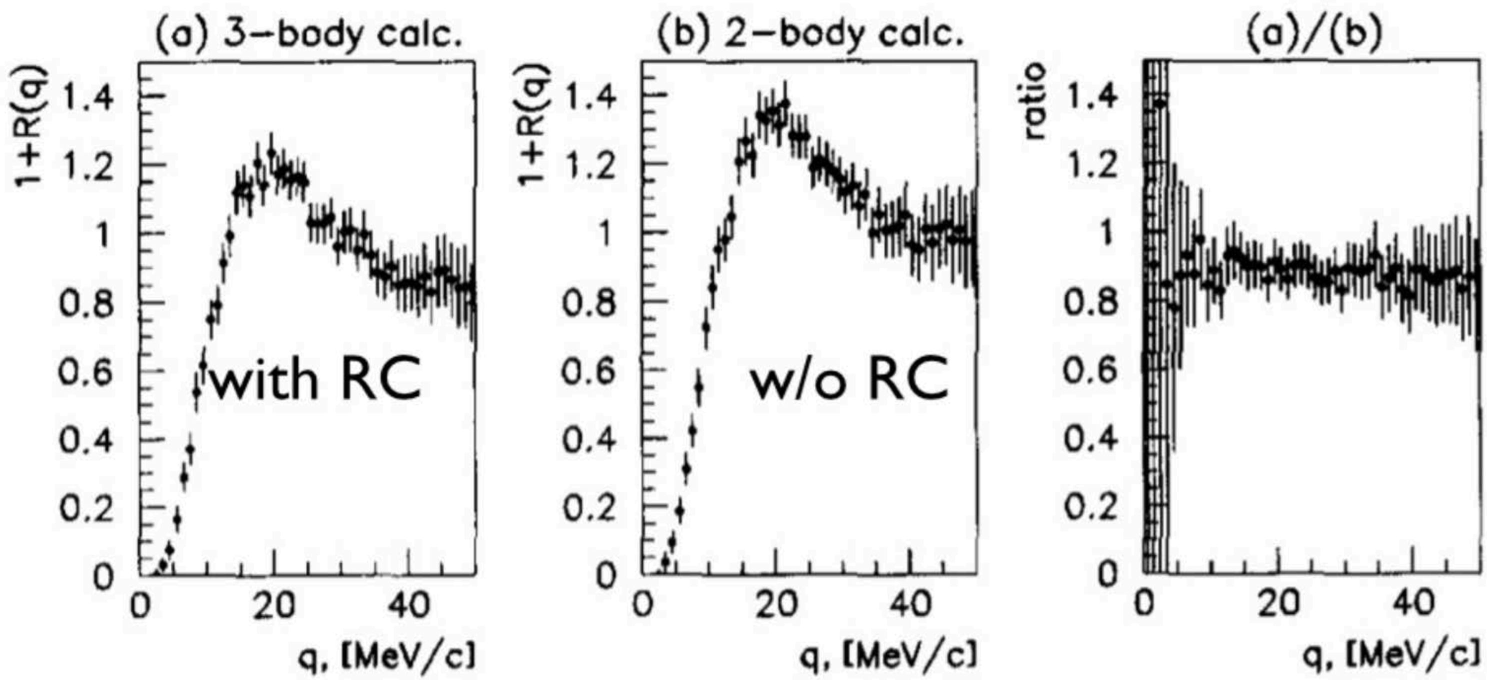
Fit $\boxed{\text{CF=Norm [Purity RQMD}(r^* \rightarrow \text{Scale}\bullet r^*)\text{+1-Purity]}}$

⇒ **RQMD overestimates r^* by 10-20% at SPS** cf **~ OK at AGS**

worse at RHIC



Residual charge



R. Lednicky, Phys. Part. Nucl. 40, 307 (2009)

B. Erazmus et al, Nucl. Phys A 583 395 (1995)

The influence of the Coulomb field of the comoving charge is included in the calculation of CF.

Residual from p-Λ correlation

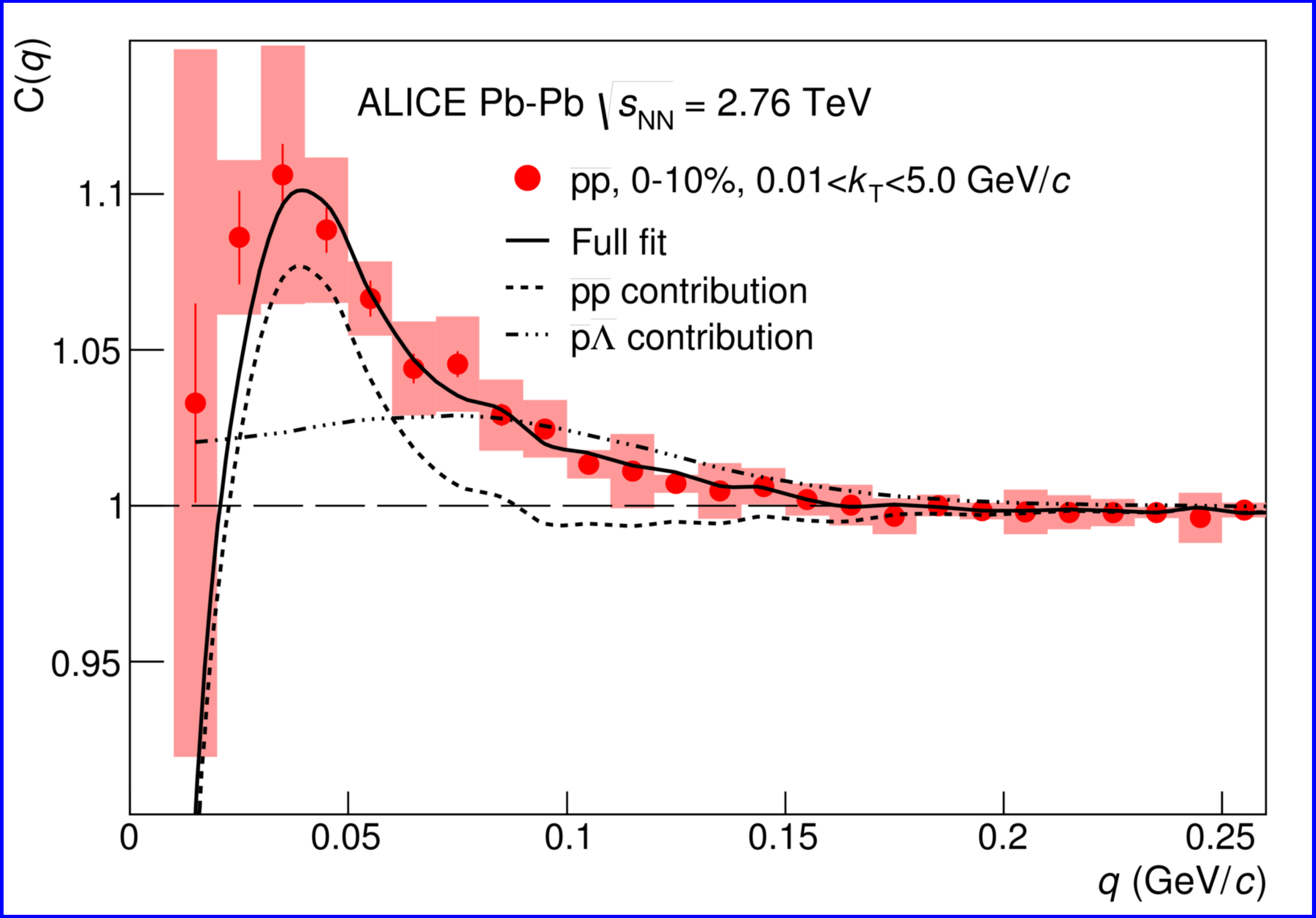
TABLE V. ΛN scattering lengths and effective ranges (in fm) and $A=4$ CSB energy differences ΔB_Λ, ΔB_Λ^* (MeV) calculated from Eqs. (46) and (47) for the potentials A, B, D, F of Nagels *et al.* (Ref. 18), and for the potential models of Sec. VII with the OPE CSB potential of Eq. (53). The errors of ΔB_Λ, ΔB_Λ^* for potentials $A-F$ are discussed in Sec. VII. ΔB_Λ^{GL} is the value calculated by Gibson and Lehman (Ref. 19). $\bar{a}_s, \bar{a}_t$ are the averages of the Λp and Λn scattering lengths and $\bar{r}_{0s}$, $\bar{r}_{0t}$ the average effective ranges. Our values of $\bar{r}_0$, obtained with Eqs. (41) and (42) for the values of $\bar{a}$ shown, are given in parentheses.

Model	$-\bar{a}_s$	$\bar{r}_{0s}$	Δa_s	$-\bar{a}_t$	$\bar{r}_{0t}$	Δa_t	ΔB_Λ^{GL}	ΔB_Λ	ΔB_Λ^*
A	2.42	2.04(3.10)	−0.51	1.17	2.43(4.50)	0.3	1.32	1.16	0.15
B	2.29	3.14(3.16)	−0.36	1.77	3.25(3.58)	0.22	0.47	0.44	−0.02
D	1.90	3.72(3.43)	−0.26	1.95	3.25(3.40)	0.22	0.43	0.38	−0.02
F	1.96	3.17(3.39)	−0.22	1.89	3.36(3.45)	0.09	0.19	0.20	−0.03
$V_{2\pi}+V_\pi^\sigma$	1.87	(3.45)	−0.09	1.89	(3.45)	0.03		0.070	−0.020
$V_{\sigma K}+V_\pi^\sigma$	1.96	(3.39)	−0.09	1.96	(3.39)	0.03		0.076	−0.019
$V_{\sigma K}+V_\pi$	1.96	(3.39)	−0.09	1.96	(3.39)	0.15		0.228	0.019

- Calculation based Lednicky and Lyuboshitz model (Sov. J. Nucl. Phys. 35 770 (1982)), with parameter-inputs from Wang & Scott (PRL 83 3138 (1999))
- Variations in the result due to the uncertainty in the input parameters have been taken into account as systematic error.

Correlation study of strong interaction

$\bar{p}\bar{p}$ & pp ALICE correlation data



Summary from pp & $\bar{p}\bar{p}$

- The first direct measurement of interaction between two antiprotons is performed by STAR. The force between two antiprotons is found to be attractive, and is as as strong as that between protons. Corresponding scattering length and effective range are found to agree with that for the force between protons.

- Besides examining CPT from a new aspect, this measurement provides a fundamental ingredient for understanding the structure of more complex anti-nuclei and their properties.