S.V. Akkelin
(BITP, Kiev)

PRINCIPAL ASPECTS OF THE THERMALIZATION PROBLEM IN HEAVY ION COLLISIONS

in collaboration with Yu.M. Sinyukov [PRC 89 (2014) 034910]

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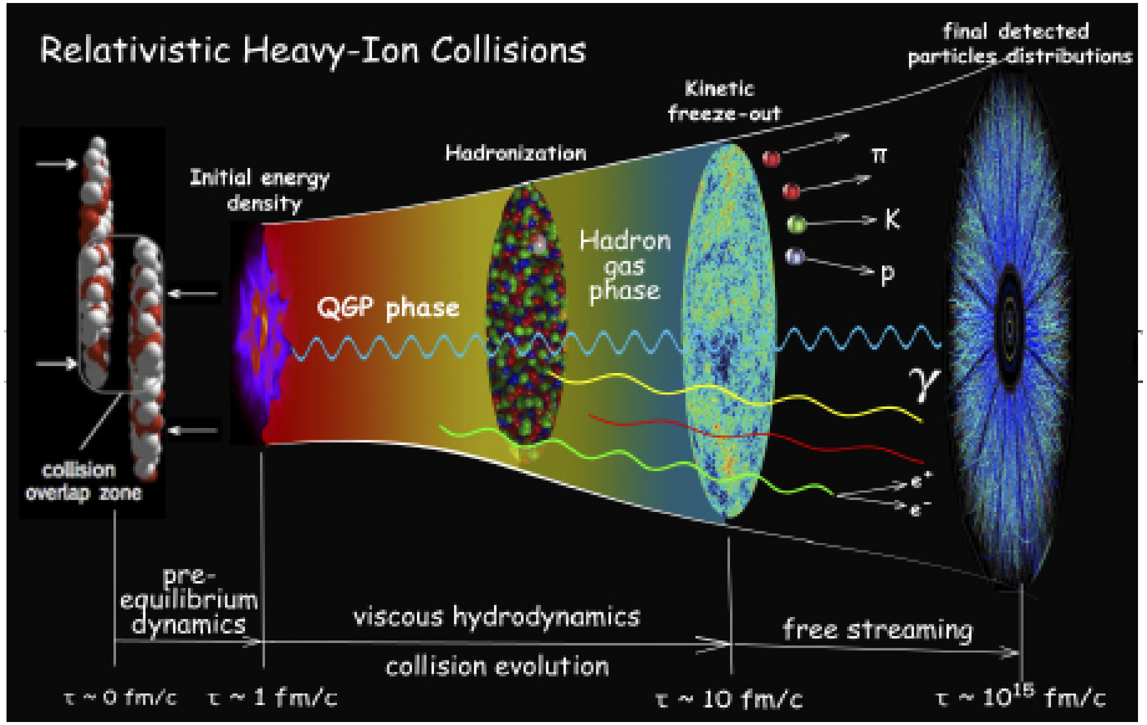
Puzzle of thermalization and entropy production in A+A collisions

Why puzzle?

System created in A+A collision demonstrates hydrodynamic-like behavior. But unitary quantum evolution of an isolated system cannot increase its entropy !

Each collision event produces (initial) pure quantum state with zero entropy, which results in (final) ensemble at observation.



Attempts to avoid the problem:
1) Classical approximation :

classical Yang-Mills equations with fluctuating initial conditions (CGC)

How classical approximation is justified ?

Main assumption: highly occupied gluon fields with small gauge coupling are produced by the statistical ensemble of classical color sources on an event-by-event basis.

Why it leads to classics?

Glauber coherent state $\hbar \to 0 \approx \alpha \to 0 \quad + \quad < H(x) > \to \infty$

Why thermalization from reversible classical dynamics?

Deterministic chaos and horizon of predictability due to extreme sensitivity to the initial conditions in complex (nonlinear) classical systems (exponential in time divergence of trajectories)

Result: non-thermal fixed point
(no equilibration at least in the classical approximation)

How to "cure" (known proposals):

Substitute irreversible dynamics (kinetics a la Boltzmann) for reversible classical dynamics when densities decrease to get irreversibility and increase of the entropy. The utilized effective kinetics is based on near-equilibrium kinetics of weakly interacting plasma, and describes the evolution of mixed state with non-zero entropy.

But: Effective kinetics (that is, in fact, some sort of coarse-grained description) is introduced "by hands" when classical approximation becomes inapplicable, and when system is far from equilibrium. If utilization of the coarse-graining procedure is unavoidable, why there is no coarse-graining from the very beginning?

OR

Add "quantum fluctuations" to reversible classical dynamics when densities decrease to generate additional chaos aiming to get irreversibility and increase of the entropy.

But: Full unitary quantum dynamics is reversible and does not change the entropy.

No first-principle thermalization?

Or something wrong with classical approximation?

Classical approximation (CA) of quantum evolution of expectation values is valid if:

1) Initial state is very specific: Glauber-like coherent state which minimizes the uncertainty relation. It is not the case in A+A collisions: initially there is a quantum superposition of different Glauber coherent states which cannot be distinguished experimentally as separate initial states of colliding nuclei!

AND

2.1) Interactions are Gaussian – CA is valid forever. It is not the case in A+A collisions.

OR

2.2) Interactions are nonlinear and quantum system has classically chaotic counterpart. Then CA is valid for restricted time interval till predictability horizon is reached. It is so because in a quantum system the unitarity of the Schrodinger evolution preserves all scalar products and, so, all the "distances" between quantum state vectors during the time, and no exponential in time divergence of "trajectories" is possible. Therefore chaos in the approximate classical dynamics of an isolated quantum system is an artefact of the approximations. It is the case in A+A collisions.

Resume:

Utilization of classical approximation for whole isolated quantum system created in A+A collision does not have robust justification. Even for appropriate initial state (it is not the case in A+A collision) and for high densities and small coupling constants CA is broken if classical (deterministic) chaos is developed (it can happen in A+A collision before the overpopulated regime is broken).

Thereby:

a) The classical chaos cannot be utilized to justify equilibration in A+A collisions.

b) The effective kinetics based on coarse-graining quasi-classical approximation can be inapplicable even in the overpopulated regime.

Attempts to avoid the problem:
2) ADS/CFT (anti-de-Sitter/conformal-field-theory) correspondence

- The AdS/CFTcorrespondence is based on the holographic gauge/string duality between four-dimensional (4D) quantum field gauge theory such as N= 4 super Yang-Mills gauge theory (which is a conformal field theory), and five-dimensional quantum string theory.
- AdS/CFT correspondence does not take place for QCD
- Calculations in dual 5D quantum string theory are possible only under some limitations, which from the QCD viewpoint means that $N_c \to \infty$ and $\lambda \to \infty$ where N_c is the number of colors and λ is the QCD coupling constant. Under such conditions a gauge/string duality is reduced to a gauge/gravity duality between 4D quantum gauge theory and 5D classical gravity theory.
- The radial coordinate r of additional spatial direction can be associated with the renormalization group energy scale (energy cutoff scale) in the gauge field theory and asymptotically high values of radius parameter correspond to gauge field theory with asymptotically high energy cutoff.

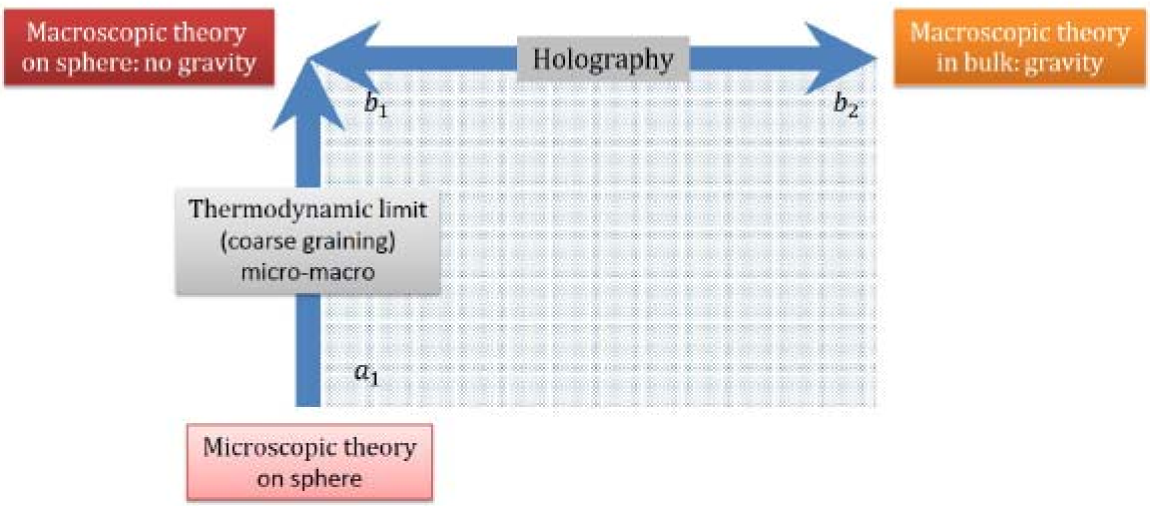
Thermalization from the ADS/CFT viewpoint

- Thermalization of 4D quantum field theory is associated with the irreversible process of black hole (and corresponding event horizon with non-zero entropy) formation.
- Long-wavelength (smoothed over short-scales) approximation of metrics induced by a large stationary black hole corresponds to a thermal state of the gauge quantum field theory.
- Long-wavelength approximation of metrics induced by a large nearly stationary black hole corresponds to a nearly perfect hydrodynamical structure of the expectation value of the energy momentum tensor of gauge quantum fields.

Question: Does ADS/CFT duality explain the thermalization in A+A collisions from the first principles ?

Answer: No

- AdS/CFT correspondence cannot be treated as the origin of thermalization and entropy creation in dual 4D quantum field theory because the latter is an ordinary quantum field theory in flat space-time, and so cannot produce the entropy in the course of reversible and unitary quantum evolution.
- Then, one can infer that the ADS/CFT thermalization is valid not for whole 4D quantum field theory but for some subsystem of it: it is well known that while entropy of the whole isolated quantum system remains constant under the time evolution, entanglement entropies of its subsystems can increase.



Resume:

Utilization of ADS/CFT correspondence for systems created in A+A collisions does not have robust justification (QCD, limits) and does not result in first-principle explanation of thermalization (ADS/CFT duality is broken in applications).

Quantum thermalization of relevant (from an observational viewpoint) subsystem in A+A collisions

Quantum entanglement and subsystems

$$t = t_0 \qquad |\Psi_{AB}\rangle = |\Psi_A\rangle \otimes |\Psi_B\rangle$$

$$t > t_0 \qquad |\Psi_{AB}\rangle = \sum_{m,n} c_{m,n} |\Psi_{A,m}\rangle \otimes |\Psi_{B,n}\rangle \neq |\Psi_A\rangle \otimes |\Psi_B\rangle$$

$$\hat{\rho}_A \otimes I_B = Tr^{(B)} |\Psi_{AB}\rangle\langle\Psi_{AB}| \qquad I_A \otimes \hat{\rho}_B = Tr^{(A)} |\Psi_{AB}\rangle\langle\Psi_{AB}|$$

$$\hat{\rho}_{AB} = |\Psi_{AB}\rangle\langle\Psi_{AB}| \qquad \hat{\rho}_{A\otimes B} = \hat{\rho}_A \otimes \hat{\rho}_B \qquad \hat{\rho}_{A\otimes B} \neq \hat{\rho}_{AB}$$

While the state of the whole system remains pure, the state of a subsystem of a composite system can be described as improper mixture represented by the partial trace of the statistical operator of the composite system in a pure state (proper mixture means incomplete knowledge for a pure state, and, typically, represents a statistical ensemble). The evolution of the corresponding reduced density matrices are governed by the non-Hamiltonian equations.

Von Neumann's entropy:

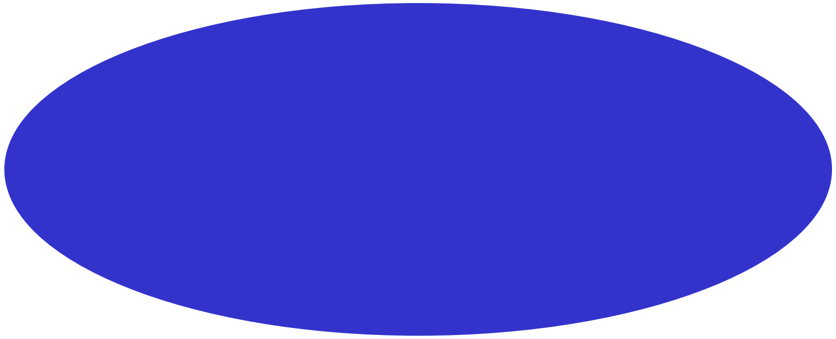
$$S_{AB} = -Tr[\hat{\rho}_{AB} \ln \hat{\rho}_{AB}]$$

$$S_A = -Tr[\hat{\rho}_A \ln \hat{\rho}_A] \qquad S_B = -Tr[\hat{\rho}_B \ln \hat{\rho}_B]$$

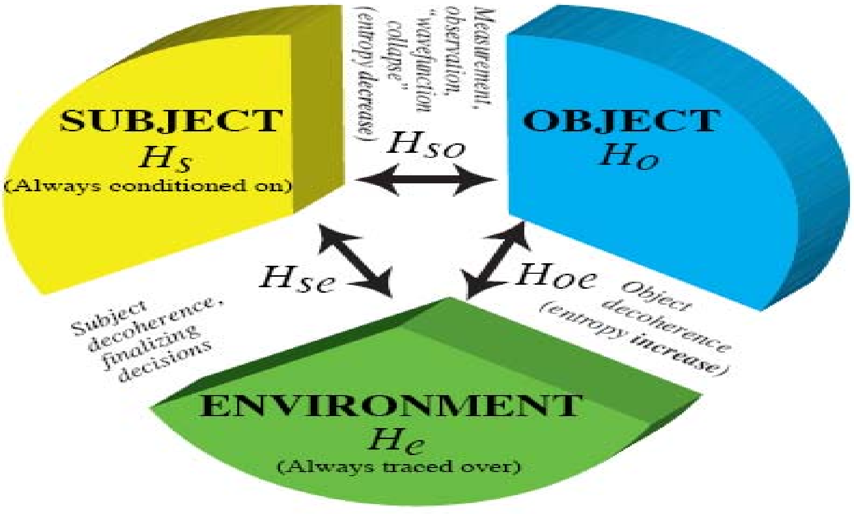
$$S_A + S_B \neq S_{AB} = 0$$

(entanglement) entropy production in an isolated quantum system can take place only after its decomposition into subsystems, and the specific way of separation of the closed system into subsystems depends on a certain experimental context (i.e., it is related with "relevant“ observables).

World without observer:
quantum non-separability
(quantum entanglement)



World with observer:
decomposition into subsystems
is context-dependent
(closed system can be resolved
into parts in various ways)



Resume:

1) Classical approximation from quantum entanglement: after decomposition into “relevant” (object) and “irrelevant” (environment) is done, an “relevant” system can be decohered (i.e., its state can be approximately diagonalized in some basis) and can acquire classical properties due to interactions with its environment containing the many degrees of freedom that are ignored from an observational point of view.

2) Thermalization is context-dependent: thermalization can arise from the description of the system by an observer who at the selected measurements and data analysis has access only to subsystem degrees of freedom, while residual degrees of freedom are entangled with the subsystem but remain unobserved. The state of the whole system, however, remains pure, and its entropy remains zero. In this kind of process, the equilibrium state of the relevant subsystem is just a state when its entropy reaches a maximum due to the build-up of entanglement of the considered system with its environment induced by the interactions.

Decoherence and approach to equilibrium of the long wavelength observables

A system created in a relativistic A+A collision can be decomposed into the fast short-length modes that represent irrelevant degrees of freedom, and slow long-length modes that represent relevant degrees of freedom. The former act as environment and can ensure decoherence and approach to (local) equilibrium for the latter. Such a splitting is conditioned by the experimental context because of limited region and accuracy in a measurement of relevant observables (e.g., particle momentum spectra) and, also, because not all possible observables are measured (e.g., not all N-particle correlations, quantum interference effects, etc.).

Scalar quantum field model: simple example

φ^4 quantum field model, whose dynamics is determined by the Lagrangian density

$$L = \frac{1}{2}\partial_\mu\varphi\partial^\mu\varphi - \frac{\lambda}{4!}\varphi^4$$

Instead of the whole system we consider the relevant subsystem, and associate the latter with long wavelength modes (i.e., with momentum scales k smaller than the some characteristic scale), then utilization of the classical approximation for expectation values of long wavelength observables can be justified, and quantum fluctuations can be accounted for short-length modes only. Quantum correlations are suppressed for the long-wavelength modes because long-wavelength mode operators are, in fact, smeared operators, and the canonical commutation relation for the smeared conjugated operators tends to zero if the scale of averaging tends to infinity.

$$\varphi = \varphi_L^{t_0} + \varphi_S^{t_0}$$

:m

The evolution equation for expectation value of long-length modes reads

$$\partial^\mu \partial_\mu \langle \varphi_L^{t_0} \rangle = -\partial^\mu \partial_\mu \langle \varphi_S^{t_0} \rangle - \frac{\lambda}{3!} \langle (\varphi_L^{t_0} + \varphi_S^{t_0})^3 \rangle$$

The initially smeared field becomes dependent on short-wavelength modes: manifestation of quantum entanglement in the Heisenberg picture !

To follow the evolution of the corresponding observables, one needs to make repeated in time splitting of the whole quantum system into the corresponding subsystems, in the Heisenberg picture this means that one needs to make repeated redefinition of the corresponding observables. The information transferred towards the irrelevant variables is discarded at the beginning of each time interval: stochastic dynamics.

Such a set of piecewise continuous evolutions can be approximated by the continuous one

$$\partial^\mu \partial_\mu \langle \varphi_L \rangle_\xi = -\frac{\lambda}{3!}(\langle \varphi_L \rangle_\xi^3 + \xi)$$

Where ξ accounts for random discontinuity of the L-modes

To get true long wavelength observables without "trembles" that are associated with different projection histories, one needs to average such observables over ξ Such an averaging means, in fact, smearing over the time interval δt for set of projection histories, and is not associated with statistical ensemble of initial events. The necessary condition for hydrodynamical approximation to be valid is the allowance to neglect, after such an averaging, non-conservation of energy momentum due to interactions with short wavelength modes, i.e., to make the assumption that such an interaction results mostly in the information loss.

The Wigner function of the long-wavelength modes:

$$N_L(x,p) =$$

$$\sum_\xi (2\pi)^{-4} \int d^4v e^{-ipv} \langle\varphi_L\rangle_\xi(x+\tfrac{1}{2}v)\langle\varphi_L\rangle_\xi(x-\tfrac{1}{2}v)$$

The energy momentum tensor of long wavelength modes:

$$T^L_{\mu\nu}(x) = \sum_\xi T_{\mu\nu}[\langle\varphi_L\rangle_\xi]$$

$$T_{\mu\nu}[\langle\varphi_L\rangle_\xi] = \int \mathrm{d}^4p \, \left(p_\mu p_\nu + \tfrac{1}{4}\partial_{x^\mu}\partial_{x^\nu} - \tfrac{1}{2}g_{\mu\nu}(p^2 + \tfrac{1}{4}\partial_x{}^2)\right) N^\xi_L(x,p) +$$

$$\frac{\lambda g_{\mu\nu}}{4!} \int \mathrm{d}^4p \mathrm{d}^4p' N^\xi_L(x,p) N^\xi_L(x,p')$$

There is no explicit contributions of the short-wavelength modes.

$$p_\mu \partial^\mu N_L(x,p) = \frac{1}{4}\partial^\mu \varrho(x) \frac{\partial}{\partial p^\mu} N_L(x,p) + \frac{1}{16}\tau^{\mu\nu}\frac{\partial}{\partial p^\mu}\frac{1}{p_0}\frac{\partial}{\partial p^\nu} N_L(x,p)$$

$$\varrho = \sum_\xi \varrho_\xi = -\frac{\lambda}{3!}\langle \varphi_L \rangle^2, \quad \sum_\xi [\delta\varrho^\mu_\xi(x)\delta\varrho^\nu_\xi(y)] = \tau^{\mu\nu}(x,y)\delta(t_x - t_y), \quad \tau^{\mu\nu} \equiv \tau^{\mu\nu}(x,x)$$

The equation describes process of momentum isotropization and spatiotemporal decoherence of the long-length modes, which precedes thermal equilibration acting on a shorter time scale and is a necessary condition for thermalization and hydrodynamics

$$N_L(x,p) \sim \exp\left[-2p_0\left(\frac{p_0 \partial^0 \varrho(x)}{\tau^{00}} + \frac{2p_i \partial^i \varrho(x)}{\tau^{ii}}\right)\right]$$

Resume:

Because the long-length modes in the initial stage of a relativistic A+A collision are highly populated, this allows us to consider evolution of the corresponding expectation value of the quantum field in the quasiclassical approximation with noise term. Such a stochasticity accounts effectively for quantum entanglement between different scales. Then, entanglement-driven stochasticity results in irreversibility and decoherence for the effective coarse-grained dynamics of the large scales.

Conclusions-1

- System created in A+A collision is not thermalized as whole and its entropy does not increase in the course of the unitary quantum dynamics.
- But: interacting quantum subsystems become entangled in the course of unitary evolution of the whole system and, as a result, their entropies increase and thermalization can happen.
- Because of quantum non-separability there is no observer-independent decomposition into subsystems.
- But: an ambiguity of a splitting procedure is removed by the requirement that separation of the closed system into subsystems must be done in a specific way depending on a certain experimental context (i.e., it is related with "relevant“ observables). One can expect that the utilization of a full unitary quantum evolution of a closed system with subsequent projection into relevant coarse-grained subspace at the measurement will result in the same predictions for a statistical ensemble of experimental data as utilization of a relevant coarse-grained effective theory that follows to instantaneous decomposition of the whole state into relevant and irrelevant subsystems/observables.

Conclusions-2

- Because observables in A+A collision are measured with some degree of precision (and not all possible observables are measured in a typical experiment), this leaves the room for inaccessible degrees of freedom. We proposed to split the system created in a relativistic A+A collision into a long-length modes subsystem and a short-length modes subsystem, and consider the former as a relevant subsystem.
- The generated non-zero entropy can be understood as the entanglement entropy of the long-length subsystem of the system created by a nucleus-nucleus collision. A fluid dynamics then appears as an effective long-wavelength theory.
- Thermalization and transition to hydrodynamics are contextual, and are related to a particular procedure of decomposition of the whole quantum system into interacting subsystems that contain a large enough number degrees of freedom. Hydrodynamical description is inappropriate for an observer who wholly measures the total set of observables for an isolated quantum system. Such an observer then will have to calculate the whole quantum evolution of a system of interest to predict results of such a "complete" experiment.